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Abstract

Contrary to the existing theories of private label products, we demonstrate that the introduction of a private label product by a retailer may improve the profits of the supplier of a competing national brand product. Our theory is built on two main elements. First, the introduction of a private label product may expand the total demand for the products carried by the retailer and thus enlarge the joint profit to be split between the retailer and the supplier of the national brand product. Second, in an environment where consumers do not know the quality of the private label product, the national brand serves as a bond to assure consumers that the retailer sells high-quality products only. This quality assurance enhances the joint profit generated by the introduction of the private label product, which, in conjunction with the weakening of the retailer's bargaining position caused by asymmetric information, may enable the national brand supplier to earn a larger profit than in the absence of the private label product.

Keywords: Private Label; Asymmetric Information; Signaling; Retailer Buyer Power

JEL Classification Numbers: L20, L15

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1 Introduction

Private labels, *i.e.*, consumer goods sold under a retailer's brand,[1] play a significant role in the retail markets of many countries. A recent study by Nielsen Company (2018) shows that in 2016 private label products accounted for 17.7 percent of retail sales in North America, and for 31.4 percent in Europe. Moreover, the significance of private labels has been growing over time. For example, Agriculture and Agri-Food Canada (2010) reports that between 2005 and 2009, private label introductions as a percentage of all product introductions increased from below 15 percent to over 20 percent in North America as well as in Europe.

The theoretical literature on this subject has examined a number of reasons for retailers to introduce private labels. Chief among them is that it strengthens a retailer's bargaining position against the associated national brand supplier, thus enabling the retailer to obtain better supply terms from the latter (Mills 1995, Narasimhan and Wilcox 1998, Bontems *et al*. 1999, Scott Morton and Zettelmeyer 2004, Caprice 2017). Another notable explanation for private labels is that they can be used as an instrument of price discrimination among heterogeneous consumers (Wolinsky 1987, Gabrielsen and Sørgard 2007).

A common thread in this literature is that while the introduction of a private label product benefits the retailer, it typically reduces the profit of the national brand supplier

[1] Private labels are also known as store brands, as they are usually sold under a retailer's own name or a name created exclusively by the retailer.

(see Bergès-Sennou, *et al*. 2004, Table 5). This arises because the private label product takes market share away from the national brand. In the extreme case, the retailer may delist the national brand product and sell the private label product only (Caprice 2017).

In this paper, we argue that the introduction of a private label product is not always detrimental to the interest of the national brand supplier. Using a theoretical model of asymmetric information, we demonstrate that there are circumstances under which the national brand supplier earns a larger profit with the introduction of the private label than without it.

Our theory is built on two main elements. First, the introduction of a private label product may enlarge the joint profit to be split between the retailer and national brand supplier. By adding a vertically differentiated private label product to its product line, the retailer may expand its market reach and attract more consumers. The resulting increase in the total quantity sold by the retailer generates a larger joint profit for the retailer and the national brand supplier.

An increase in joint profit by itself, however, does not necessarily entail a larger profit for the national brand supplier because the introduction of private label product weakens its bargaining position relative to the retailer. Indeed, as mentioned above, the literature has shown that the introduction of private label product typically reduces the profit of the national brand supplier. This conclusion, however, is obtained from models in which all agents have perfect information.

This brings us to the second main element in our theory, which is asymmetric information about product quality. To be more specific, we consider an environment where consumers know the quality of a national brand product but do not know the quality of a private label product.[2] In such an environment, the retailer may have an incentive to misrepresent the quality of the private label product. This moral hazard problem weakens the retailer's ability to use the private label as a bargaining tool against the national brand supplier. Moreover, the presence of the national brand in the retailer's product line helps alleviate the moral hazard problem by serving as a bond to assure consumers that the private label product is of high quality. Both factors counteract the strengthening of the retailer's bargaining position brought about by the introduction of private label product.

The combination of these two elements implies that the national brand supplier may earn a larger profit with the introduction of the private label product than without it. The first element enlarges the joint profit, thus creating the potential for the supplier's profit to go up. The asymmetric information enables the national brand supplier to wrestle a share of the increased joint profit from the retailer.

To demonstrate our theory, we construct and analyze a model in which a retailer sells one or both of the following two products, a national brand and a private label, over two

[2] In the literature, Bergès-Sennou and Waterson (2005) is – to our knowledge – the only other theoretical analysis that considers asymmetric information about the quality of private label product. Their focus is different from the present paper in that they investigate the factors that determine a retailer's decision to introduce a reputable private label product. They find, among other things, that for products purchased infrequently, introducing a reputable private label is not sustainable in equilibrium.

periods. The products are vertically differentiated, with the national brand product having the highest quality. The quality of private label product, chosen by the retailer, has two possible levels, high or low. Consumers know the quality of a national brand product. But they do not know the true quality of the private label product when it is introduced in the first period; they find out its quality in the second period only if some consumers have consumed it in the previous period.

We show that in this model there are three possible motives for the retailer to introduce the private label product. The first motive is to replace the national brand with the private label product. This occurs in equilibrium if the unit cost of producing the national brand product is relatively high in comparison with the quality gap between the two products and the unit cost of the private label product. The second motive is to use the private label product as a bargaining tool to obtain better supply terms from the national brand supplier. This motive, which has been extensively explored in the literature (Mills 1995, Bontems *et al*. 1999, Bergès-Sennou 2006, Meza and Sudhir 2010, Caprice 2017), arises in our model if the unit cost of the national brand product is relatively low.

The third motive, which has not received much attention in the literature, is to expand the total demand for the products carried by the retailer. This motive exists in the case where the unit cost of the national brand product falls into an intermediate range. In this case, the launch of the private label product not only improves the retailer's bargaining position, but also expands the joint profit of the retailer and the national brand supplier.

Consistent with the literature, we find that the introduction of the private label product always reduces the profit of the national brand supplier if consumers have perfect information about the quality of the private label product. This is true even in the case where the addition of the private label product expands the joint profit of the two firms.

In comparison with the equilibrium under perfect information, asymmetric information about the quality of the private label product benefits the national brand supplier in a number of ways. First, it expands the range of parameter values over which the national brand is carried by the retailer. Second, in cases where both products are sold it increases the quantity of the national brand and decreases the quantity of the private label product. Third, it lowers the profit the retailer would earn in the event that it fails to reach an agreement with the national brand supplier, thus reducing the effectiveness of the private label as the retailer's bargaining tool.

Most interestingly, the presence of asymmetric information enables the national brand supplier to earn a larger profit with the introduction of the private label product than without it under certain conditions. In our model, the national brand serves as a bond to assure consumers that the retailer sells the high-quality private label product.[3] This quality assurance enhances the profit generated by the introduction of the private label product, which, in conjunction with the weakening of the retailer's ability to use the private label as a bargaining tool, may enable the national brand supplier to earn a larger

[3] Specifically, we assume that if the retailer misrepresents the quality of the private label product in the first period, it will ruin its reputation and lose all of its customers, including those who would have purchased the national brand product, in the second period.

profit than in the absence of the private label product. Our analysis shows that this will indeed happen if the quality of the national brand product is sufficiently high.

The rest of this paper is organized as follows. We describe the model in section 2. In section 3, we analyze the equilibrium in a benchmark case of perfect information, and we present our findings for the case of asymmetric information in section 4. We conclude in section 5.

2. Model

2.1. The firms

We consider a model where a retailer sells vertically differentiated products. In addition to a national brand (NB) product, it has the option of selling a private label (PL) product. The latter has two possible levels of quality: a high-quality PL product or a low-quality PL product. Let s_B, s_H and s_L denote the quality levels of NB, high-quality PL and low-quality PL product, respectively. We assume that $s_B > s_H > s_L$; in other words, national brand is of higher quality than a private label product.[4]

In the upstream market, the retailer purchases the NB product from a monopoly supplier who produces it at a constant marginal cost c_B. In the event that the retailer chooses to offer a PL product, it can procure the good from a supplier selected from a pool of competitive contract manufacturers. Competition among these suppliers enables

[4] This assumption is common in the literature on private labels (Mills 1995, Narasimhan and Wilcox 1998, Bontems *et al*. 1999, Scott Morton and Zettelmeyer 2004, Avenel and Caprice 2006, Gabrielsen and Sørgard 2006). As observed by Steiner (2004), while the quality of PL products has improved over time, on average their quality is still lower than that of the national brands.

the retailer to procure the product at its marginal cost, denoted by c_H for a unit of high-quality PL and c_L for a unit of low-quality PL. We assume that marginal cost of production rises with quality; hence, $c_B > c_H > c_L$. To simplify presentation, we set $c_L = 0$.

In order to launch a private label, the retailer has to incur a sunk cost, denoted by k, to design and develop the product. We assume that k is infinitesimal so that it serves only as a tiebreaker in the case where the retailer would otherwise be indifferent between offering and not offering a PL product.

The retailer sets the retail price of the NB product and (where applicable) the retail price of the PL product, denoted by p_B (NB product), p_H (high-quality PL product) and p_L (low-quality PL product), respectively.

In the upstream market, the wholesale price of the NB product is determined through negotiation between the NB supplier and the retailer. To be more precise, suppose that the contract between the supplier and the retailer takes the form of two-part tariff $T = (w, F)$, where w denotes the wholesale price of the NB product and F denotes a fixed fee charged to the retailer. We use the Generalized Nash Bargaining solution (Harsanyi and Selten, 1972) to model the negotiation between the retailer and the NB manufacturer.

2.2. Consumers

Consumers have heterogeneous preferences over quality. Following the classic quality-choice model of Mussa and Rosen (1978), we assume that consumers' preferences over quality are uniformly distributed over an interval $[0, \bar{\theta}]$ with density

$1/\bar{\theta}$. Specifically, let $\theta^j \in [0, \bar{\theta}]$ denote the location of a typical consumer j. She receives a net surplus

$$u^j = s_i \theta^j - p_i, \qquad (1)$$

if she purchases a product of quality s_i at price p_i, where $i \in \{B, H, L\}$. If she purchases none of the three good, her surplus is zero. In this paper, we want to consider a situation where the low-quality PL product is of such poor quality that a consumer will not (knowingly) purchase it at any positive price. For this purpose, we assume $s_L = 0$.[5]

From (1), it is straightforward to derive the demand functions for the products in various scenarios. In particular, let Q_B (respectively, Q_H) denote the quantity of the NB product (respectively, high-quality PL product) sold by the retailer. In a scenario where the retailer sells only one product, NB ($i = B$) or high-quality PL ($i = H$), the demand function for the product is:

$$Q_i = \max\left\{\bar{\theta} - \frac{p_i}{s_i}, 0\right\}, \quad (i = B, H). \quad (2)$$

On the other hand, if the retailer chooses to sell both NB and high-quality PL products, the demand functions for these two products are

$$Q_B = \max\left\{\bar{\theta} - \frac{p_B - p_H}{s_B - s_H}, 0\right\} \text{ and } Q_H = \max\left\{\frac{p_B - p_H}{s_B - s_H} - \frac{p_H}{s_H}, 0\right\}. \quad (3)$$

In (2) and (3), we take into account the possibility that the retail demand for a good could be zero if its price is too high.

Note that for the demand in (2) to be positive, $\bar{\theta}$ has to be sufficiently large.

[5] More generally, the results from our analysis will continue to hold for a positive s_L as long as it is sufficiently small that $\theta s_L \leq c_L$.

Accordingly, we assume that

$$\bar{\theta} > \max\left\{\frac{c_B}{s_B}, \frac{c_H}{s_H}\right\}. \qquad (4)$$

This assumption ensures that when the NB product (respectively, the high-quality PL product) is priced at marginal cost, the demand for this good is positive.

Another way of looking at condition (4) is that, for a given value of $\bar{\theta}$, the marginal cost of producing the NB product (respectively, the high-quality PL product) cannot be too large. To be more exact, define $c_B^a \equiv s_B\bar{\theta}$ and $c_H^\alpha \equiv s_H\bar{\theta}$. Then (4) can be rewritten as $c_B < c_B^a$ and $c_H < c_H^\alpha$. The entire analysis in this paper is conducted under these assumptions on c_B and c_H.

2.3. Information Structure

An important factor we will consider in this analysis is the observability of product quality. Specifically, we assume that the PL product is an experience good, so that consumers do not know its quality when the product is introduced in period 1, and they learn the quality in period 2 either from their own consumption experience or through word of mouth (*i.e.*, the passing of product information via consumer-to-consumer communications).[6] The retailer and the NB supplier, on the other hand, have perfect information about the quality of the PL product.[7] Moreover, the quality of the NB

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[6] We will have more discussions about this assumption in section 4.

[7] We suppose that the NB supplier is able to acquire information about the quality through the process of negotiation with the retailer. One of the reasons for the retailer to launch a PL product is to strengthen its bargaining power *vis-à-vis* the supplier. To achieve this, the retailer will have to present credible evidence to convince the NB supplier that it has indeed developed the PL product. As a manufacturer, the NB supplier will be able to ascertain the quality of the product from the evidence presented by the retailer.

product is common knowledge.

As a benchmark, we will also examine the equilibrium under the alternative assumption of perfect information about product quality. In this alternative scenario, consumers know the quality of the PL product before they purchase it. We will analyze this scenario in section 3 and study the case of asymmetric information in section 4.

2.4. Timing

Subject to the information structure described above, firms and consumers play a game over two periods. The game in the first period unfolds in three stages. At stage one, the retailer chooses the products it plans to carry in the two periods. To be more specific, it selects from the following three possible product lines, indexed by (NB, PL), (NB, 0), and (0, PL). They represent, respectively, the product line with (i) both the NB and the PL products, (ii) only the NB product, and (iii) only the PL product. In cases where the retailer chooses a product line that includes the private label, it also determines the quality of the product.

At stage two, if the retailer has chosen to carry the NB product as part of its product line, it negotiates a contract with the NB supplier that covers both period 1 and period 2. In the event that the two firms fail to reach an agreement, the retailer would sell only the PL product if it has incurred the cost of developing the PL product at stage one. Otherwise, its disagreement payoff would be zero. In either case, the disagreement payoff of the NB supplier is zero.

At stage three, the retailer sets the retail prices of the product(s) in period 1 and period 2. After observing these prices, consumers make their purchase decisions for period 1. If the retailer carries the PL product, consumers will make their decision based on their belief about its quality.

In period 2, consumers make their purchase decisions based on their updated belief about product quality. The period 2 payoff of each player is discounted by a factor $\delta \in (0,1)$.

3. Equilibrium under Perfect Information

This section analyzes the equilibrium under the assumption that players have perfect information. In particular, consumers know the quality of the PL product before they purchase it. This equilibrium will serve as a benchmark for the analysis of the model under asymmetric information.

In this model of perfect information, we use the solution concept of subgame perfect equilibrium. Accordingly, we start with an analysis of the subgames associated with each of the following three product lines: (0, PL), (NB, 0) and (NB, PL).

The first observation we can make about the equilibrium under perfect information is that the retailer will not choose to develop a PL product of low quality. This is because the assumption $s_L = 0$ implies that consumers will not knowingly buy a low-quality PL product at any positive price. Since the retailer has to incur a cost k to develop a PL product, it is not profitable for the retailer to develop a low-quality PL product under perfect information. Therefore, we can rule out the low-quality PL product as a part of an equilibrium product line under perfect information.

3.1. Product Line (0, PL)

Consider the subgame after the retailer chooses the product line (0, PL). In this subgame, the retailer will sell the PL product only. Accordingly, there is no need for the retailer to negotiate with the NB supplier at stage two of the game. Instead, it moves straight to stage three where it sets the retail prices of the PL product in period 1 and period 2.

Recall that the cost of developing the private label (k) is a sunk cost incurred by the retailer at stage one. Accordingly, it has no effect on the retailer's decisions in a subgame starting from stage two. Rather, the retailer's decision will be based on its gross profit without deducting k or, to be more precise, its quasi-rent.

As noted above, the retailer will not sell a low-quality PL product under perfect information. Let p_H^t denote the retail price of the high-quality PL product in period t (=1, 2). The retailer's total (discounted) quasi-rent over the two periods is given by

$$\Pi_H(p_H^1, p_H^2) = (p_H^1 - c_H)\left(\bar{\theta} - \frac{p_H^1}{s_H}\right) + \delta(p_H^2 - c_H)\left(\bar{\theta} - \frac{p_H^2}{s_H}\right). \quad (5)$$

The retailer chooses p_H^1 and p_H^2 to maximize (5). Solving this optimization problem, we find the equilibrium prices of the PL product:

$$p_H^1 = p_H^2 = p_H^*, \qquad \text{where } p_H^* \equiv \frac{s_H\bar{\theta}}{2} + \frac{c_H}{2}. \qquad (6)$$

Substituting p_H^* into the demand function for the PL product, we obtain

$$Q_H^1 = Q_H^2 = Q_H^*, \qquad \text{where } Q_H^* \equiv \frac{\bar{\theta}}{2} - \frac{c_H}{2s_H}. \qquad (7)$$

Note that condition (4), or equivalently $c_H < c_H^{\alpha}$, ensures that the quantity of the PL product given in (7) is positive. Substituting (6) into (5), we obtain the retailer's maximum quasi-rent associated with product line (0, PL):

$$\Pi_H^* = \frac{(1+\delta)(s_H\bar{\theta} - c_H)^2}{4s_H}. \tag{8}$$

Note that the preceding analysis is also relevant to the subgame associated with the product line (NB, PL). If the retailer chooses to offer this product line, it will negotiate with the NB supplier at stage two of the game about the terms under which it purchases the NB product. In the event that it fails to reach an agreement with the NB supplier, it would sell to consumers the PL product only. Equation (8) represents the profit that the retailer would receive in that scenario. In other words, (8) is the retailer's disagreement payoff in the subgame associated with the product line (NB, PL).

3.2. Product Line (NB, 0)

Now suppose the retailer has chosen the product line (NB, 0) at stage one. At stage three of period 1 and in period 2, the retailer sells the NB product only. Let (w^t, F^t) denote the wholesale price and fixed fee paid by the retailer to the NB supplier in period t (=1, 2). The retailer's total (discounted) quasi-rent over the two periods is:

$$\Pi_B(p_B^1, p_B^2) = \left[(p_B^1 - w^1)\left(\bar{\theta} - \frac{p_B^1}{s_B}\right) - F^1\right] + \delta\left[(p_B^2 - w^2)\left(\bar{\theta} - \frac{p_B^2}{s_B}\right) - F^2\right], \tag{9}$$

The retailer chooses the retail prices (p_B^1, p_B^2) to maximize (9), which yields

$$p_B^t = \frac{s_B\bar{\theta}}{2} + \frac{w^t}{2}, \quad (t = 1, 2). \tag{10}$$

At stage two, the terms of contract between the retailer and the NB supplier is determined by the Generalized Nash Bargaining solution. This has two implications for the equilibrium values of (w^t, F^t). First, the retailer and the NB supplier will agree to choose the wholesale prices that maximize their joint profit. Given the presence of double marginalization problem (Spengler, 1950), it is easy to show that their joint-surplus is maximized at $w^1 = w^2 = c_B$.

Second, the terms of contract must satisfy the solution to the Generalized Nash Bargaining Problem. To be more specific, let Π_B^* denote the maximum joint quasi-rent associated with product line (NB, 0), π_R the retailer's payoff, and π_M the NB supplier's payoff under the terms of contract. The retailer's disagreement payoff in this case is 0 because it has not developed the private label. Therefore, the Generalized Nash Bargaining Problem is written as:

$$\max_{\pi_R, \pi_M} {\pi_R}^{\omega} \pi_M^{1-\omega} \quad \text{s.t. } \pi_R + \pi_M = \Pi_B^*. \qquad (11)$$

In (11), $\omega \in (0,1)$ is a parameter that measures the retailer's bargaining power relative to that of the NB supplier. Solving (11) we find the familiar Nash barraging solution:

$$\pi_R = \omega \Pi_B^*, \qquad \pi_M = (1-\omega)\Pi_B^*. \qquad (12)$$

The retailer and the NB supplier can achieve the division of joint quasi-rent specified in (12) by setting the appropriate levels of the fixed fees.

Setting $w^1 = w^2 = c_B$ in (10), we obtain the equilibrium retail price of the NB product in this subgame:

$$p_B^1 = p_B^2 = p_B^*, \qquad \text{where } p_B^* \equiv \frac{s_B \bar{\theta}}{2} + \frac{c_B}{2}. \qquad (13)$$

Substituting p_B^* into the demand function yields

$$Q_B^1 = Q_B^2 = Q_B^*, \qquad \text{where } Q_B^* \equiv \frac{\bar{\theta}}{2} - \frac{c_B}{2 s_B}. \qquad (14)$$

Note that condition (4), or equivalently $c_B < c_B^a$, ensures that the quantity of the NB product given in (14) is positive. Using (13) and (14), we find the maximum joint quasi-rent over the two periods to be:

$$\Pi_B^* = \frac{(1+\delta)(s_B\bar{\theta} - c_B)^2}{4s_B}. \qquad (15)$$

The equilibrium payoffs of the retailer and the NB supplier can be obtained by substituting (15) into (12).

3.3. Product Line (NB, PL)

Finally, consider the subgame associated with the product line (NB, PL). From (3), we see that the demand functions for the NB product and the PL product in period t (=1, 2) are

$$Q_B^t(p_B^t, p_H^t) = \bar{\theta} - \frac{p_B^t - p_H^t}{s_B - s_H}, \quad Q_H^t(p_B^t, p_H^t) = \frac{p_B^t - p_H^t}{s_B - s_H} - \frac{p_H^t}{s_H}. (16)$$

Note that, while the retailer has chosen to carry both products at stage one, it may set the retail price of one of these products at such a high level that the demand for this good is 0. Taking into account this possibility, we write the retailer's optimization problem regarding retail prices as:

$$\max_{p_B^t, p_H^t} \Pi_{BH} = \sum_{t=1}^{2} \delta^{t-1}[(p_B^t - w^t)Q_B^t(p_B^t, p_H^t) + (p_H^t - c_H)Q_H^t(p_B^t, p_H^t) - F^t], (17)$$

$$s.t.\ Q_B^t(p_B^t, p_H^t) \geq 0; Q_H^t(p_B^t, p_H^t) \geq 0. \qquad (18)$$

The constraints in (18) are needed to prevent the retailer from choosing those prices that would make Q_B^t or Q_H^t in (16) negative.

Let λ_1^t and λ_2^t be the multipliers attached to these constraints in the Lagrange function associated with this optimization problem. Solving the first-order conditions of this optimization problem, we find

$$p_B^t = \frac{s_B\bar{\theta}}{2} + \frac{w^t}{2} - \frac{\lambda_1^t}{2}; \qquad (19)$$

$$p_H^t = \frac{s_H\bar{\theta}}{2} + \frac{c_H}{2} - \frac{\lambda_2^t}{2}. \qquad (20)$$

We see from (19) and (20) that the retail prices depend on the values λ_1^t and λ_2^t, which in turn hinge on whether any of the constraints in (18) is binding.

In the Generalized Nash Bargaining Problem at stage two, the retailer would be able to sell a positive quantity of the PL product in the event of failure to reach an agreement with the NB supplier. Accordingly, the retailer's disagreement payoff is given by Π_H^* in (8). Let Π_{BH}^* denote the maximum joint quasi-rent associated with product line (NB, PL). It is then straightforward to derive the Nash barraging solution in this case:

$$\pi_R = \omega\Pi_{BH}^* + (1-\omega)\Pi_H^*, \qquad \pi_M = (1-\omega)\Pi_{BH}^* - (1-\omega)\Pi_H^*. \quad (21)$$

The complete analysis of this subgame is long and tedious. Hence, we relegate the details of the analysis to an appendix. Here we present only a summary of the results associated with this subgame.[8]

Lemma 1. Under perfect information, the quantity of each product sold in the equilibrium of the subgame associated with the product line (NB, PL) depends on the value of c_B relative to the following critical values:

$$c_B^a \equiv s_B\bar{\theta}, \qquad c_B^b \equiv (s_B - s_H)\bar{\theta} + c_H, \qquad c_B^c \equiv \frac{s_B c_H}{s_H}, \qquad (22)$$

with $c_B^a > c_B^b > c_B^c$ $(> c_H)$. To be more specific:

[8] The proofs of all lemmas and propositions are also in the appendix.

a) In the case where $c_B \in (c_H, c_B^c]$, $Q_B^t > 0$ and $Q_H^t = 0$ for $t = 1$ and 2, that is, the quantity of national brand sold is positive while that of the private label is 0 in both periods.

b) In the case where $c_B \in (c_B^c,\ c_B^b)$, $Q_B^t > 0$ and $Q_H^t > 0$ for $t = 1$ and 2, that is, the quantities sold are positive for both the national brand and the private label in both periods.

c) In the case where $c_B \in [c_B^b,\ c_B^a)$, $Q_B^t = 0$ and $Q_H^t > 0$ for $t = 1$ and 2, that is, quantity of the national brand sold is 0 while that of the private label is positive in both periods.

Lemma 1 states that the retailer will sell positive quantities of both the NB product and the PL product if the unit cost of the national brand is in an intermediate range, namely, $c_B^c < c_B < c_B^b$. On the other hand, if the unit cost is high in the sense that $c_B \geq c_B^b$, no units of the NB product will be sold in equilibrium. Finally, if the unit cost is relatively low ($c_B \leq c_B^c$), the retailer will not sell any unit of the PL product even though it has developed this product at stage one of the game.

Note that the critical values c_B^b and c_B^c defined in (22) depend on the level of c_H and the relative magnitudes of s_B and s_H. They imply that the occurrence of each scenario in Lemma 1 as an equilibrium depends on the unit cost of NB product relative to that of the (high-quality) PL product and the quality gap between the two products. For example, scenario c) in Lemma 1 occurs if the unit cost of NB product is high relative to the unit cost of the PL product and relative to the quality gap between the two products (*i.e.*, if $c_B \geq c_H s_B / s_H$).

3.4. Subgame Perfect Equilibrium

At stage one of the game, the retailer compares its quasi-rents associated with the three product lines while taking into consideration the cost of developing the PL product k. It then chooses the product line that yields the highest profit. We will start with a discussion of the equilibrium product line, followed by a number of observations about other aspects of the equilibrium.

Proposition 1. Under perfect information, the retailer's choice of product line in equilibrium depends on the magnitude of c_B. Specifically,

a) In the case where $c_B \in (c_H, c_B^c]$, the equilibrium product line is (NB, PL) with $Q_B^t > 0$ and $Q_H^t = 0$ for $t = 1$ and 2.

b) In the case where $c_B \in \left(c_B^c, c_B^b\right)$, the equilibrium product line is (NB, PL) with $Q_B^t > 0$ and $Q_H^t > 0$ for $t = 1$ and 2.

c) In the case where $c_B \in \left[c_B^b, c_B^a\right)$, the equilibrium product line is (0, PL).

Proposition 1 reflects three motives for the retailer to launch the PL product. The first motive is to replace the NB product with the PL product. This occurs if the unit cost of the NB product is so high that $c_B \geq c_B^b$, in which case the retailer drops the NB from its product line.

The second motive is to use the PL product to strengthen its bargaining position. This motive is behind the equilibrium in the case where the unit cost of the NB product is so low that $c_B \in (c_H, c_B^c]$. In this case, the retailer develops the PL but does not sell any quantity of this product in equilibrium. The PL product is used solely for the purpose of enhancing the retailer's bargaining position. This motive for launching private labels has been extensively studies in the literature (Mills, 1995; Bontems *et al.*, 1999; Bergès-Sennou, 2006; Meza and Sudhir 2010).

The third motive, which has not received much attention in the literature, is to expand the total demand for the products carried by the retailer. This motive exists for c_B in the intermediate case, $c_B \in \left(c_B^c, c_B^b\right)$. In this case, the retailer finds it most profitable to carry and sell a positive quantity of both products. While the launch of the PL product improves the retailer's bargaining position, this is not the only motive for the retailer to sell this product. In fact, developing and selling the PL product is profitable in its own right, as implied by the following proposition.

Proposition 2. Suppose $c_B \in \left(c_B^c, c_B^b\right)$. Under perfect information, the combined quantity of the national brand and private label products sold in equilibrium is larger than the quantity that would have been sold if the retailer had chosen the product line (NB, 0). Moreover, the joint profit of the two firms and the profit of the retailer are larger than those if the retailer had chosen the product line (NB, 0).

Proposition 2 states that with the launch of the PL product, the retailer is able to sell more units. Recall that each consumer buys one unit of a good. A larger quantity sold means that the retailer has expanded its market reach and sold to more consumers. This, in turn, increases the joint profit of the retailer and the NB supplier.

Proposition 2 suggests an interesting possibility that the NB supplier might actually earn a larger profit when the retailer sells the PL product. While the NB supplier's share of the joint profit declines after the retailer launches the PL product, the resulting increase in joint profit might still raise the amount of profit received by the supplier. It turns out, however, this is not true in the equilibrium under perfect information.

Proposition 3. Suppose $c_B \in \left(c_B^c, c_B^b\right)$. Under perfect information, the equilibrium profit of the NB supplier associated with (NB, PL) is lower than that if the retailer had offered the product line (NB, 0).

Propositions 2 and 3 indicate that under perfect information, the introduction of the PL product enlarges the joint profit of the retailer and the NB supplier, but the additional profit is reaped by the retailer rather than shared with the supplier. Intuitively, there are two forces at play here. The first one is the strengthening of the retailer's bargaining position brought about by the PL product. This force reduces the supplier's profit. The second force is the increase in joint profit, which tends to offset the reduction in profit for the supplier. But the magnitude of the second force is dominated by the first one. Hence, the supplier's profit falls.

4. Equilibrium under Asymmetric Information

In this section, we analyze the equilibrium under the original assumption that the PL product is an experience good, in which case a consumer does not know the quality of the PL product when it is introduced in period 1 and she learns its quality in period 2 either from her own consumption experience or through word of mouth. In this analysis, we suppose that the word-of-mouth effect is so widespread that, if the PL product is sold in period 1 to some consumers, the information about its quality is disseminated to all consumers in period 2.[9]

The asymmetric information about the quality of the PL product implies that the retailer may have an incentive to cheat and misrepresent a low-quality PL product as a high-quality one. Accordingly, we need to consider the retailer's incentive compatibility constraint. This constraint may affect the retailer's choice of product line and prices in equilibrium.

Critical to the incentive compatibility constraint is how consumers would respond in period 2 if the retailer cheats in period 1. There are a number of possible ways to model the consumers' response in this situation. For example, we could assume that in period 2 consumers would continue to shop at this retailer (with the knowledge that the PL product is of low quality) in period 2 despite its dishonesty in the previous period. Alternatively, we could assume that the retailer's cheating behavior ruins its reputation among its customers, causing some or all of them to stay away from the retailer in period 2. In other words, consumers would punish the retailer for its cheating behavior.

In this analysis, we assume that all consumers would boycott the retailer in period 2 if it cheats in period 1. This represents the most severe punishment that consumers could impose on the retailer. This assumption captures an important element in our theory,

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[9] The word-of-mount (WOM) effect on sales of consumer products is well documented in the marketing literature (see Marchand *et al.* 2017, and the literature cited therein). With the expansion of electronic commerce and social media, the WOM effect is becoming increasingly significant as more and more consumers use the Internet to share their experience with the products and services they have purchased.

namely, the NB product may be used as a bond to ensure that the retailer sells the high-quality PL product. Under this assumption, the retailer who sells low-quality PL product in period 1 loses all of its sales of the NB product in period 2.[10]

We use perfect Bayesian equilibrium (PBE) as the solution concept for this model of incomplete information. An important element of a PBE is the players' belief system, specifically, the consumers' beliefs about the quality of the PL product. We assume that each consumer formulates her belief in the following way. Let ρ denote a consumer's subjective probability that the PL product is of high quality. If the retailer offers a product line that contains the private label in period 1, the consumer will believe that the product is of high quality (*i.e.*, $\rho = 1$) if and only if the prices chosen by the retailer satisfy the incentive compatibility (IC) constraint for the high-quality PL product. The IC constraint requires that the retailer's payoff from offering the high-quality PL product be no lower than that from selling the low-quality PL product disguised as a high-quality one. If the IC constraint is not satisfied, any claim by the retailer about the PL product being of high-quality will not be credible and hence will not be believed by consumers, in which case $\rho = 0$.

Similar to the case of perfect information, we start with an analysis of the three continuation games after the retailer's choice of product line at stage 1. Since consumers

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[10] An alternative way to capture this idea of the national brand product serving as a bond is to assume that the NB supplier would sever its relationship with the retailer in period 2 if the latter cheats in period 1. This could arise if the supplier is concerned about damages to its own reputation from dealing with a retailer that misrepresents a low-quality product as a high-quality one.

observe the product line chosen by the retailer, we can analyze the equilibrium in the continuation game associated with each of the three product lines, (0, PL), (NB, 0) and (NB, PL), separately.

Note that asymmetric information about the quality of the PL product affects only the product lines (0, PL) and (NB, PL). The product line (NB, 0) does not involve the PL product. Hence, the equilibrium in the continuation game associated with this product line does not change as we move from perfect information to asymmetric information. Therefore, we only need to reconsider the equilibria in the continuation games associated with the product lines (0, PL) and (NB, PL).

4.1 Product Line (0, PL)

As noted earlier, a consumer will not knowingly purchase a low-quality PL product at price above 0. The retailer, for its part, will have no incentive to incur the cost k to develop the PL product if it expects that it will not be able to sell it at a price above 0 for at least one period. Consequently, the retailer will launch the private label only if it can convince the consumers, in period 1, that the product is of high quality. To do so, the retail prices have to satisfy the IC constraint.

The preceding discussion implies that a retailer will never choose the low-quality PL product in equilibrium. For the low-quality PL product to be profitable, it has to be sold at a price above 0 for at least one period. In order to convince consumers to purchase the PL product at a price above 0, the retailer has to satisfy the IC constraint. However, if the IC constraint is satisfied, the retailer will have no incentive to choose the low-quality product. Therefore, if the retailer chooses a product line that contains the private label in equilibrium, the product has to be of high quality.

In the case where the retailer has chosen the product line (0, PL), the IC constraint takes the following form:

$$(p_H^1 - c_H)Q_H^1 + \delta(p_H^2 - c_H)Q_H^2 \geq p_H^1 Q_H^1. \qquad (23)$$

The left-hand side of (23), to be denoted by $\Pi_H(p_H^1, p_H^2)$, is the retailer's quasi-rent if it sells the high-quality PL product at price p_H^1 in period 1 and p_H^2 in period 2. The right-hand side is the retailer's quasi-rent if it sells a low-quality PL product and manages to mislead some consumers into buying the product at the price p_H^1 in period 1. But it will not be able to sell any unit of the PL product in period 2 as its low quality becomes known to all consumers. Therefore, (23) requires that the retailer's payoff from offering the high-quality PL product be at least as high as that from selling the low-quality product disguised as the high-quality product.

At stage three of the game associated with the product line (0, PL), the retailer chooses (p_H^1, p_H^2) to maximize its quasi-rent $\Pi_H(p_H^1, p_H^2)$ subject to the IC constraint (23). Working with the first-order conditions of this constrained optimization problem, we find the equilibrium in this continuation game depends on the value of c_H. Specifically, let

$$c_H^\gamma = \frac{\delta s_H \bar{\theta}}{2+\delta}. \qquad (24)$$

Recalling that $c_H^\alpha = s_H\bar{\theta}$, we can see from (24) that $c_H^\gamma < c_H^\alpha$.

Lemma 2 Under asymmetric information, the retailer sells a positive quantity of high-quality PL product and earns a positive profit from the product line (0, PL) as long as $c_H < c_H^{\alpha}$. Moreover,

a) If $c_H \leq c_H^{\gamma}$, the retailer chooses the same price, sells the same quantity of the PL product and earns the same level of profit as under perfect information, that is, $p_H^1 = p_H^2 = p_H^*$, $Q_H^1 = Q_H^2 = Q_H^*$ and $\Pi_H = \Pi_H^*$.

b) If $c_H^{\gamma} < c_H < c_H^{\alpha}$, the retailer chooses a higher price, sells a smaller quantity of the PL product and earns a lower level of profit in period 1 than under perfect information. Its price, quantity and profit in period 2 are at the same levels as those under perfect information. To be more specific, $p_H^1 = \hat{p}_H \equiv p_H^* + \lambda c_H/2$, $p_H^2 = p_H^*$; $Q_H^1 = \hat{Q}_H \equiv Q_H^* - \lambda c_H/2s_H$, $Q_H^2 = Q_H^*$ and $\Pi_H = \hat{\Pi}_H \equiv \Pi_H^* - \lambda^2 c_H^2/4s_H$, where

$$\lambda = \frac{(2+\delta)(s_H\bar{\theta} - c_H)(c_H - c_H^{\gamma})}{2c_H^2} > 0. \quad (25)$$

Lemma 2 states that if the unit cost of the high-quality PL product is sufficiently low $\left(c_H \leq c_H^{\gamma}\right)$, the equilibrium in the continuation game associated with the product line (0, PL) is not affected by asymmetric information. In this scenario, the IC constraint is slack and hence the retailer faces no temptation to mislead consumers about the quality of the PL product. But the temptation to cheat becomes stronger if the marginal cost is higher (*i.e.*, if $c_H > c_H^{\gamma}$). In this scenario, the retailer has to set its price in period 1 above that under perfect information (*i.e.*, $\hat{p}_H > p_H^*$) to satisfy the IC constraint. This reduces the quantity of PL product sold in period 1 and leads to a lower level of retail profit than under perfect information (*i.e.*, $\hat{Q}_H < Q_H^*$ and $\widehat{\Pi}_H < \Pi_H^*$).

Note that the higher equilibrium price ($\hat{p}_H > p_H^*$) in the case $c_H \in \left(c_H^\gamma,\ c_H^\alpha\right)$ serves to signal the quality of PL product. Intuitively, the retailer's temptation to cheat stems from the higher profit it would earn from selling the low-quality PL product in period 1. By raising the price of the PL product above the level that maximizes its profit under perfect information, the retailer sells fewer units in period 1. This, in turn, reduces its one-period gain from selling the low-quality product. In other words, by restricting the quantity of PL product sold in period 1, the retailer increases the relative importance of future profits (in period 2), thus sending a credible signal that the PL product is of high quality.

Another observation from Lemma 2 is that the price and quantity of the PL product in period 2 are the same as that under perfect information. This is because once the PL product is sold in period 1, all consumers learn its quality in period 2, allowing the retailer to choose the same price as the one that maximizes its profit under perfect information.

Recall that (0,PL) is the product line that the retailer would end up if it chooses (NB,PL) at stage one but fails to reach an agreement with the NB supplier at stage two of the game. Part b) in Lemma 2 suggests that for $c_H > c_H^\gamma$, the presence of asymmetric information decreases the retailer's disagreement payoff, thus weakening its bargaining position *vis-à-vis* the NB supplier.

4.2 Product Line (NB, PL)

With the product line (NB, PL), the retailer sells both the NB product and the PL product. As noted in section 4.1, the retailer will not choose the low-quality PL product in equilibrium. Therefore, we only need to analyze the situation where the private label in the product line (NB, PL) is of high quality.

To state the incentive compatibility constraint in this case, let Π_{BL} denote the retailer's quasi-rent if it misrepresents the low-quality product as the high-quality one. Then,

$$\Pi_{BL} = (p_B^1 - w^1)Q_B^1(p_B^1, p_H^1) + p_H^1 Q_H^1(p_B^1, p_H^1) - F^1. \qquad (26)$$

Note in (26) that the retailer does not earn any profit in period 2. This is because the retailer would not be able to sell any unit of either the PL product or the NB product. Recall from (17) that Π_{BH} is the retailer's quasi-rent if it sells the high-quality PL product along with the NB product. The IC constraint in this case can then be written as $\Pi_{BH} \geq \Pi_{BL}$.

As in the case of perfect information, the retailer may set the prices in such a way that the demand for one of the products is 0 in equilibrium. With those observations in mind, we write the retailer's optimization problem at stage two as:

$$\max_{p_B^t, p_H^t} \Pi_{BH} = \sum_{t=1}^{2} \delta^{t-1}[(p_B^t - w^t)Q_B^t(p_B^t, p_H^t) + (p_H^t - c_H)Q_H^t(p_B^t, p_H^t) - F^t], (27)$$

$$s.t. \ \Pi_{BH} \geq \Pi_{BL}; Q_B^t(p_B^t, p_H^t) \geq 0; Q_H^t(p_B^t, p_H^t) \geq 0. \quad (28)$$

At stage two of the game, the retailer and the NB supplier negotiate the terms of contract (w^1, F^1, w^2, F^2). Recall that the disagreement point of this bargaining problem is that the retailer sells the PL product only. Hence, the payoffs given in Lemma 2 represent the retailer's disagreement payoffs for different values of c_H.

Because of the presence of the IC constraint, the analysis of the equilibrium in the continuation game associated with this product line under asymmetric information is even more intricate than that under perfect information. To conserve space, we relegate all details of this analysis to the appendix.[11] Here we will make one general observation about this case. As in the case of the product line (0, PL), the presence of asymmetric information would have no impact on the equilibrium if the unit cost of the PL product is sufficiently low, specifically if $c_H \leq c_H^\gamma$. In this case, the retailer has no incentive to cheat on the quality of the PL product and thus the IC constraint is slack. If the unit cost of PL product is higher (*i.e.*, if $c_H > c_H^\gamma$), however, the IC constraint may become binding and consequently the equilibrium price and quantity of the PL product in period 1 may be different from their counterparts under perfect information.

4.3 Perfect Bayesian Equilibrium

Based on the analysis of the retailer's payoffs associated with the three product lines, we determine its choice of product line at stage one and the resulting perfect Bayesian equilibrium. It turns out that in the case where $c_H \leq c_H^\gamma$, the equilibrium product line and quantities under asymmetric information are the same as those under perfect information

[11] In particular, see Lemma A in the appendix for a description of the equilibrium in this continuation game.

(as presented in Proposition 1). Since our main interest here is in the differences made by asymmetric information, we omit the discussions of the PBE associated with $c_H \le c_H^\gamma$ and focus, instead, on the range of c_H in the interval $\left(c_H^\gamma, c_H^\alpha\right)$.

To present the equilibrium product line for c_H in this range, we need to define another critical value of c_B. Let.

$$c_B^{b'} = s_B\overline{\theta} - \frac{\delta(s_H\overline{\theta} - c_H)^2}{2c_H}. \qquad (29)$$

It can be shown that $c_B^c < c_B^b < c_B^{b'} < c_B^a$ for c_H in the range (c_H^γ, c_H^α).

Proposition 4. Suppose $c_H \in \left(c_H^\gamma,\ c_H^\alpha\right)$. The product line and quantities sold in a perfect Bayesian equilibrium under asymmetric information depends on the magnitude of c_B as follows.

a) If $c_B \in (c_H, c_B^c]$, the equilibrium product line is (NB, PL), but the quantity of the private label product is zero in both periods.

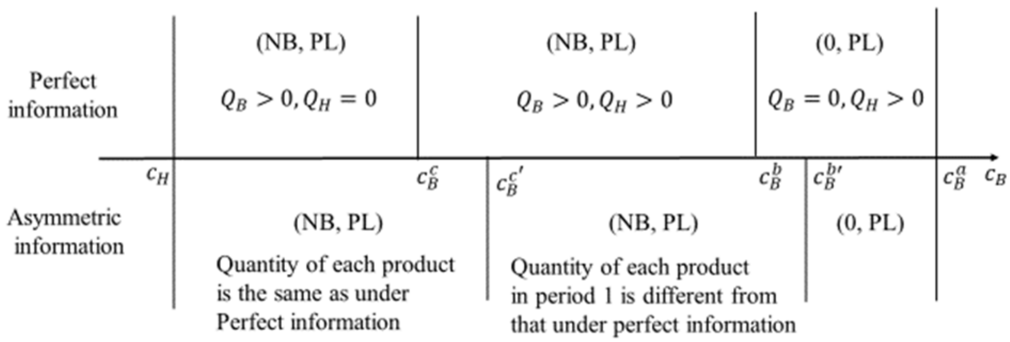
b) If $c_B \in \left(c_B^c, c_B^b\right)$, the equilibrium product line is (NB, PL), and the quantities of both products are positive in both periods.

c) If $c_B \in \left[c_B^b, c_B^{b'}\right)$, the equilibrium product line is (NB, PL), the quantities of both products are positive in period 1, but the quantity of the national brand product falls to 0 in period 2.

d) If $c_B \in \left[c_B^{b'}, c_B^a\right)$, the equilibrium product line is (0, PL).

A comparison Proposition 4 with Proposition 1 shows that the retailer has the same types of motives to introduce PL product under asymmetric information as under perfect information. Specifically, the retailer uses the PL product to strengthen its bargaining position in the case where the unit cost of the NB product is relatively low (*i.e.*, $c_B \leq c_B^c$). If the unit cost of the NB product is relatively high (*i.e.*, $c_B \geq c_B^{b'}$), on the other hand, the retailer uses the PL product to replace the NB product. In the case where the unit cost of the NB product is in the intermediate range, the launch of the PL product enables the retailer to expand the total demand for the goods it carries.

Figure 1: Equilibrium Product Line and Quantity



Note: The equilibrium under asymmetric information illustrated here is for the case $c_H \in (c_H^\gamma, c_H^\alpha)$.

One notable difference between the equilibria under asymmetric information and perfect information can be seen in part c) of Propositions 4. For $c_B \in [c_B^b, c_B^{b'})$, the retailer sells positive quantities of both PL product and NB product in period 1, but it stops selling the NB product in period 2. What happens in this case is that the retailer uses the NB product to assure consumers that the PL product is of high quality in period 1 and then sells the PL product exclusively once its quality becomes public information in period 2. Recall from part c) of Proposition 1 that under perfect information the retailer does not carry the NB product for c_B in this range. Therefore, as illustrated in Figure 1, the presence of asymmetric information expands the range of c_B over which the NB product is offered, from $\left(c_H, c_B^b\right)$ to $\left(c_H, c_B^{b'}\right)$.

The impact of asymmetric information goes beyond the expanded range of parameter values for which the national brand is offered in equilibrium. It also affects the quantity of each product sold under different product lines. As noted above, for c_B in the interval $\left[c_B^b, c_B^{b'}\right)$, the retailer sells a positive quantity of the NB product in period 1 under asymmetric information, while it would have sold no NB product at all under perfect information. Moreover, asymmetric information may influence the quantity of each product for c_B in the interval $\left(c_B^c, c_B^b\right)$ as well. Specifically, define another critical value of c_B,

$$c_B^{c'} = c_B^b + \frac{c_H}{\delta} - \frac{\sqrt{\Omega}}{\delta s_H}, \qquad (30)$$

where $\Omega = s_H^2[\delta\bar{\theta}(s_B - s_H) + (1+\delta)c_H]^2 - \delta s_B s_H[(2+\delta)c_H^2 + \delta s_H \bar{\theta}^2(s_B - s_H)]$. As illustrated in Figure 1, this critical value of c_B divides the interval $\left(c_B^c, c_B^b\right)$ into two segments. It turns out that if $c_B \in (c_B^c, c_B^{c'})$, the IC constraint is slack and the equilibrium quantities of both products in the product line (NB, PL) under asymmetric information are the same as those under perfect information. But if $c_B \in (c_B^{c'}, c_B^b)$, the IC constraint is binding and the equilibrium quantity of the PL product is smaller and that of the NB product is larger (in period 1) under asymmetric information than those under perfect information. More generally, we have the following proposition.

Proposition 5. Suppose $c_H \in \left(c_H^\gamma,\ c_H^\alpha\right)$. The quantity of the private label product sold is no larger, and the quantity of the national brand product sold is no smaller, under asymmetric information than that under perfect information. In particular, the quantity of the private label product sold under asymmetric information is smaller than that under perfect information if $c_B \in \left(c_B^{c'}, c_B^{\alpha}\right)$. The quantity of the national brand product sold under asymmetric information is larger than that under perfect information if $c_B \in \left(c_B^{c'},\ c_B^{b'}\right)$.

The changes in the quantities of products sold under asymmetric information also have an impact on the profits of the two firms. Generally speaking, the presence of asymmetric information tends to reduce the profit of the retailer but raise the profit of the NB supplier.

Proposition 6. Suppose $c_H \in \left(c_H^{\gamma},\ c_H^{\alpha}\right)$. The retailer's profit is lower under asymmetric information than under perfect information. On the other hand, the supplier of national brand earns a larger profit under asymmetric information than under perfect information if $c_B \in \left(c_H,\ c_B^{b'}\right)$.

Intuitively, the retailer's profit is lower for two reasons. First, asymmetric information reduces the retailer's disagreement payoff and thus weakens its ability to use the PL product as a bargaining tool. Second, the need to satisfy the incentive compatibility constraint distorts the prices and quantities and thus decreases the joint profit for the two firms when a positive quantity of the PL product is sold in equilibrium. The latter occurs for $c_B \in \left(c_B^{c'},\ c_B^{\alpha}\right)$.

Proposition 6 indicates that the asymmetric information benefits the NB supplier for a wide range of parameter values. Even in situations where the presence of asymmetric information reduces the joint profit, the effect of a smaller pie is overwhelmed by the increase in the size of the supplier's slice, enabling it to earn a larger profit than under perfect information.

Having examined the equilibrium under asymmetric information in comparison with that under perfect information, we now zoom in on the case where the retailer chooses the product line (NB, PL) and investigate how the introduction of the PL product affects the quantities and profits under asymmetric information.

Proposition 7. Suppose $c_H \in \left(c_H^{\gamma},\ c_H^{\alpha}\right)$ and $c_B \in (c_B^{c}, c_B^{b'})$. Under asymmetric information, the combined quantity of the national brand and private label products sold in equilibrium is larger than the quantity that would have been sold if the retailer had chosen the product line (NB, 0). Moreover, the joint profit of the two firms and the profit of the retailer are larger than those if the retailer had chosen the product line (NB, 0).

For the range of c_H and c_B specified in Proposition 7, the equilibrium product line is (NB, PL) and the retailer sells a positive quantity of the PL product in both periods. Under such circumstances, the presence of the PL product in the product line increases the total quantity (of two products) sold and raises the joint profit of the retailer and the NB supplier. While this result also holds under perfect information (see Proposition 2), its implication for the profit of the NB supplier may be different under asymmetric information.

In particular, asymmetric information makes it possible that the supplier of the NB product earns a larger profit when the PL product is sold than if it had not been sold. To describe the conditions under which this occurs, we define one more critical value of c_B:

$$c_B^{c\dagger} = c_B^c + \frac{(s_H\bar{\theta} - c_H)}{2s_H c_H}\sqrt{\frac{\delta s_B(s_B - s_H)}{1+\delta}[2(2+\delta)c_H s_H\bar{\theta} - \delta c_H^2 - \delta s_H^2\bar{\theta}^2]}. \qquad (31)$$

In (31), the expression under the square root is positive for $c_H \in \left(c_H^{\gamma}, c_H^{\alpha}\right)$. It is clear from (31) that $c_B^{c\dagger} > c_B^c$.

Proposition 8. Suppose $c_H \in \left(c_H^{\gamma},\ c_H^{\alpha}\right)$ and $c_B \in \left(c_B^c, c_B^{b'}\right)$. Under asymmetric information, there exist $S > 0$ and $c_B^{b\dagger} \in (c_B^{c'}, c_B^{b'})$ such that for $s_B > S$ and $c_B \in \left(c_B^{c\dagger}, c_B^{b\dagger}\right)$, the supplier of the national brand earns a larger profit with the equilibrium product line (NB, PL) than with the alternative product line (NB, 0).

Proposition 8 delivers the most interesting result in this paper, that is, the introduction of the PL product raises, rather than lowers, the profit of the NB supplier under some circumstances. Specifically, this occurs for a range of c_B if the quality of the product (as measured by s_B) is sufficiently high. Intuitively, the high quality of the NB product enables the retailer to charge a high price and earn a large profit from this product. Under asymmetric information, this helps the retailer to assure consumers about the quality of the PL product: if the retailer misleads consumers in period 1, it will suffer a significant loss of profit in period 2 when consumers stop purchasing both products from the retailer. This heavy reliance on the NB product weakens the retailer's ability to use the PL product as a bargaining tool against the NB supplier, thus enabling the latter to wrestle a larger share of the joint profit from the retailer. Therefore, an interesting implication of this proposition is that suppliers of national brands are more likely to benefit from the introduction of private label products if their products are of much higher quality.

5. Conclusions

We have studied the impact of introducing a private label product under asymmetric information on the profits of a retailer and the supplier of a competing national brand product. In particular, we have demonstrated that the introduction of the private label product by the retailer is not always detrimental to the interest of the national brand supplier. This conclusion is built on two premises. First, the introduction of a private label product expands the total demand for the products carried by the retailer and thus enlarges the joint profit to be split between the two firms. Second, in an environment where consumers do not know the quality of the private label product, the retailer has an incentive to misrepresent its quality. This moral hazard problem weakens the retailer's ability to use the private label as a bargaining tool against the national brand supplier. Moreover, the presence of the national brand in the retailer's product line helps alleviate the moral hazard problem by serving as a bond to assure consumers that the private label product is of high quality. The quality assurance provided by the presence of the national brand product enhances the profit generated by the introduction of the private label product. This, in conjunction with the weakening of the retailer's ability to use the private label product as a bargaining tool, may enable the national brand supplier to earn a larger profit than if the private label product had not been introduced. An interesting implication of our findings is that suppliers of national brands are more likely to benefit from the introduction of private label products if their products are of much higher quality.

To keep our model tractable, we have made a number of simplifying assumptions. Among these assumptions, two are particularly notable. First, the game lasts only two periods, and second, the quality of the private label product becomes known to all consumers in period 2 after it is sold to some consumers in period 1. These assumptions are clearly unrealistic and they lead to the result that the retailer needs to signal the quality of the private label product for one period only. In reality, a product may be sold for many periods and new consumers may enter the market every period. Consequently, at any given time there may be some consumers who are not informed about the quality of a retailer's private label product. Accordingly, there may be a need for the retailer to signal its quality in every period. It would be an interesting extension to our analysis to consider a multi-period model with overlapping cohorts of consumers. It is our conjecture that the theory presented in this paper would continue to hold in such a model. But the verification of this conjecture is left for future research.

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Appendix

Proof of Lemma 1: The Kuhn-Tucker conditions associated with the constrained optimization problem (17)-(18) are:

$$\bar{\theta}(s_B - s_H) - 2p_B^t + 2p_H^t - c_H + w^t - \lambda_1^t + \lambda_2^t = 0, \quad (A1)$$
$$2s_H p_B^t - s_H w^t - 2s_B p_H^t + s_B c_H + s_H \lambda_1^t - s_B \lambda_2^t = 0, \quad (A2)$$
$$Q_B^t(p_B^t, p_H^t) \geq 0; Q_H^t(p_B^t, p_H^t) \geq 0; \lambda_1^t \geq 0; \lambda_2^t \geq 0; \lambda_1^t Q_B^t = 0; \lambda_2^t Q_H^t = 0. \quad (A3)$$

Solving (A1) and (A2), we obtain (19) and (20). Setting $w^1 = w^2 = c_B$ in (19) and (20), we rewrite the equilibrium prices as:

$$p_B^t = \frac{s_B\bar{\theta}}{2} + \frac{c_B}{2} - \frac{\lambda_1^t}{2}, p_H^t = \frac{s_H\bar{\theta}}{2} + \frac{c_H}{2} - \frac{\lambda_2^t}{2}. \qquad (A4)$$

Substituting (A4) into (16), we find the equilibrium quantities:

$$Q_B^t = \frac{\bar{\theta}}{2} - \frac{c_B - c_H - \lambda_1^t + \lambda_2^t}{2(s_B - s_H)}, \quad Q_H^t = \frac{c_B - c_H - \lambda_1^t + \lambda_2^t}{2(s_B - s_H)} - \frac{c_H - \lambda_2^t}{2s_H}. \quad (A5)$$

Since the demand for each product is the same over two periods under perfect information, $\lambda_1^1 = \lambda_1^2$ and $\lambda_2^1 = \lambda_2^2$.

Before we prove each of the three parts of Lemma 1, note that $c_B^a > c_B^b > c_B^c > c_H$ can be verified using their definitions.

In part a) of this lemma, $Q_B^t > 0$ and $Q_H^t = 0$. Complementary slackness conditions in (A3) requires that $\lambda_1^t = 0$ and $\lambda_2^t \geq 0$. Setting $Q_H^t = 0$ in (A5), we find

$$\lambda_2^t = \frac{s_B c_H - s_H c_B}{s_B}. \qquad (A6)$$

Then $\lambda_2^t \geq 0$ implies that $c_B \leq c_B^c$. Moreover, $c_B < c_B^a$ ensures that $Q_B^t > 0$. Therefore, we have $Q_B^t > 0$ and $Q_H^t = 0$ for $c_B \in (c_H, c_B^c]$.

In part b) of this lemma, both Q_B^t and Q_H^t are positive. Then (A3) requires that $\lambda_1^t = 0$ and $\lambda_2^t = 0$. Substituting $\lambda_1^t = 0$ into (A5), we confirm that $Q_B^t > 0$ and $Q_H^t > 0$ for $c_B \in (c_B^c,\ c_B^b)$. For later analysis, note that the equilibrium joint profit in this case is

$$\Pi_{BH}^* = \frac{(1+\delta)\{s_H(s_B\bar{\theta} - c_B)[\bar{\theta}(s_B - s_H) - c_B + c_H] + (s_H\bar{\theta} - c_H)(s_H c_B - s_B c_H)\}}{4s_H(s_B - s_H)}. \qquad (A7)$$

In part c) of this lemma, $Q_B^t = 0$ and $Q_H^t > 0$. Accordingly, (A3) requires that $\lambda_1^t \geq 0$, $\lambda_2^t = 0$. Setting $Q_B^t = 0$ in (A5), we derive $\lambda_1^t = -\bar{\theta}(s_B - s_H) + c_B - c_H$, from which we verify that $\lambda_1^t \geq 0$ if and only if $c_B \geq c_B^b$. Moreover, using (A5) we confirm that $Q_H^t > 0$ under the assumption $c_H < c_H^a$. Therefore, we have $Q_B^t = 0$ and $Q_H^t > 0$ for $c_B \in [c_B^b, c_B^a)$. Q.E.D.

Proof of Proposition 1: First, consider the case of $c_B \in (c_H, c_B^c]$. The retailer's profits associated with the product line (NB, 0), (0, PL) and (NB, PL) are $\omega\Pi_B^*$, $\Pi_H^* - k$ and $\omega\Pi_B^* + (1-\omega)\Pi_H^* - k$, respectively. The product line (NB, PL) is more profitable than (NB, 0) since $(1-\omega)\Pi_H^* - k > 0$ for a small k. The product line (NB, PL) is more profitable than (0, PL) if

$$\Pi_B^* - \Pi_H^* = \frac{(1+\delta)(s_B\bar{\theta} - c_B)^2}{4s_B} - \frac{(1+\delta)(s_H\bar{\theta} - c_H)^2}{4s_H} > 0. \quad (A8)$$

Using (A8) we find that $\partial(\Pi_B^* - \Pi_H^*)/\partial c_B < 0$ for $c_B \in (c_H, c_B^c]$. When c_B is at the point c_B^c,

$$\Pi_B^* - \Pi_H^* = \frac{(1+\delta)s_B(s_B\bar{\theta} - c_B)^2}{4s_H^2} > 0. \quad (A9)$$

This implies that $\Pi_B^* - \Pi_H^* > 0$ for all $c_B \in (c_H, c_B^c]$, suggesting that the product line (NB, PL) is more profitable than (0, PL) for c_B in this interval. Therefore, the retailer earns the highest profit from the product line (NB, PL) when $c_B \in (c_H, c_B^c]$. By Lemma 1, the quantity of the PL product is zero for c_B in this range.

In the case of $c_B \in (c_B^c, c_B^b)$, the retailer's profits associated with product line (NB, 0), (0, PL) and (NB, PL) are $\omega\Pi_B^*$, $\Pi_H^* - k$ and $\omega\Pi_{BH}^* + (1-\omega)\Pi_H^* - k$, respectively. The product line (NB, PL) is more profitable than (NB, 0) if $\omega(\Pi_{BH}^* - \Pi_B^*) + (1-\omega)\Pi_H^* > 0$. Since

$$\Pi_{BH}^* - \Pi_B^* = \frac{(1+\delta)(s_H c_B - s_B c_H)^2}{4(s_B - s_H)s_B s_H} > 0, \quad (A10)$$

we have $\omega\Pi_{BH}^* + (1-\omega)\Pi_H^* > \omega\Pi_B^*$, which implies that the product line (NB, PL) is more profitable than (NB, 0) (for a small k). Moreover, the product line (NB, PL) is more profitable than (0, PL) because

$$\Pi_{BH}^* - \Pi_H^* = \frac{(1+\delta)[\bar{\theta}(s_B - s_H) - c_B + c_H]^2}{4(s_B - s_H)s_B} > 0. \quad (A11)$$

Therefore, the retailer can earn the highest profit from the product line (NB, PL) for $c_B \in \left(c_B^c, c_B^b\right)$, and the quantities of both products are positive since $c_B^c < c_B < c_B^b$.

In the case of $c_B \in \left[c_B^b, c_B^a\right)$, the retailer's profits associated with product line (NB, 0), (0, PL) and (NB, PL) are $\omega\Pi_B^*$, $\Pi_H^* - k$ and $\Pi_H^* - k$, respectively. Between (NB, PL) and (0, PL) the retailer would choose the latter because the retailer does not gain any additional profit from carrying the NB product. Moreover, the product line (0, PL) is more profitable than (NB, 0) if $\Pi_H^* - \omega\Pi_B^* - k \geq 0$. It can be shown that

$$\Pi_H^* - \Pi_B^* = \frac{(1+\delta)(s_H\bar{\theta} - c_H)^2}{4s_H} - \frac{(1+\delta)(s_B\bar{\theta} - c_B)^2}{4s_B} \quad (A12)$$

is monotonically increasing for $c_B \in \left[c_B^b, c_B^a\right)$. At point $c_B = c_B^b$,

$$\Pi_H^* - \Pi_B^* = \frac{(1+\delta)(s_B - s_H)(s_H\bar{\theta} - c_H)^2}{4s_B s_H} > 0. \quad (A13)$$

This implies that $\Pi_H^* - \Pi_B^* > 0$ for all $c_B \in \left[c_B^b, c_B^a\right)$, suggesting that the product line (0, PL) is more profitable than (NB, 0) for $c_B \in \left[c_B^b, c_B^a\right)$. Therefore, the retailer earns the highest profit from the product line (0, PL) when $c_B \in \left[c_B^b, c_B^a\right)$. Q.E.D.

Proof of Proposition 2: In the case of $c_B \in \left(c_B^c, c_B^b\right)$, the equilibrium product line is (NB, PL) and the quantities of both goods are positive. Setting $\lambda_1^t = \lambda_2^t = 0$ in (A5), we obtain the combined quantity of these two goods in equilibrium:

$$Q_{B|BH}^* + Q_{H|BH}^* = \frac{\bar{\theta}}{2} - \frac{c_H}{2s_H}. \quad (A14)$$

If the retailer chooses the product line (NB, 0), the equilibrium quantity (of the NB product) is

$$Q_B^* = \frac{\bar{\theta}}{2} - \frac{c_B}{2s_B}. \quad (A15)$$

Since $c_B > c_B^c$, from (A14) and (A15) we find that $Q_{B|BH}^* + Q_{H|BH}^* > Q_B^*$; that is, the combined quantity of (NB, PL) is larger than the quantity of (NB, 0).

Moreover, the joint profits of the two firms from the product line (NB, PL) and (NB,0) are Π_{BH}^* and Π_B^*, respectively. According to Proposition 1 (case b), $\Pi_{BH}^* > \Pi_B^*$ for $c_B \in \left(c_B^c, c_B^b\right)$. The retailer's profits from the product line (NB, PL) and (NB, 0) are $\omega\Pi_{BH}^{*}+(1-\omega)\Pi_{H}^{*}-k$ and $\omega\Pi_{B}^{*}$, respectively. Since $\Pi_{BH}^{*}>\Pi_{B}^{*}$, we conclude that $\omega\Pi_{BH}^{*}+(1-\omega)\Pi_{H}^{*}-k>\omega\Pi_{B}^{*}$ for a small k; in other words, the retailer's profit is larger with the product line (NB, PL) than with (NB, 0). Q.E.D.

Proof of Proposition 3: In the case of $c_B\in\left(c_B^c,c_B^b\right)$, the profits that the NB supplier earns from the product line (NB, PL) and (NB, 0) are $(1-\omega)\Pi_{BH}^{*}-(1-\omega)\Pi_{H}^{*}$ and $(1-\omega)\Pi_{B}^{*}$, respectively. The former is lower than the latter if $\Pi_{B}^{*}-(\Pi_{BH}^{*}-\Pi_{H}^{*})>0$. It can be verified that

$$\Pi_{B}^{*}-(\Pi_{BH}^{*}-\Pi_{H}^{*})=\frac{(1+\delta)[\bar{\theta}^2 s_B s_H(s_B-s_H)-s_H c_B^2-s_B c_H^2+2s_B c_B c_H-2s_B c_H\bar{\theta}(s_B-s_H)]}{4s_B(s_B-s_H)}\quad (A16)$$

is monotonically decreasing for $c_B\in\left(c_B^c,c_B^b\right)$. At $c_B=c_B^b$, $\Pi_{B}^{*}-(\Pi_{BH}^{*}-\Pi_{H}^{*})=(1+\delta)(s_H\bar{\theta}-c_H)^2/4s_B>0$. This implies that $\Pi_{B}^{*}-(\Pi_{BH}^{*}-\Pi_{H}^{*})>0$ for all $c_B\in\left(c_B^c,c_B^b\right)$. Therefore, the profit of the NB supplier associated with the product line (NB, PL) is lower than that if the retailer had offered the product line (NB, 0) in the case of $c_B\in\left(c_B^c,c_B^b\right)$. Q.E.D.

Proof of Lemma 2: Let λ denote the Lagrange multiplier associated with the IC constraint (23). The Kuhn-Tucker conditions associated with the maximization of the retailer's quasi-rent $\Pi_H(p_H^1,p_H^2)$ (subject to the IC constraint (23)) are:

$$s_H\bar{\theta}-2p_H^1+c_H+\lambda c_H=0,\quad (A17)$$
$$(1+\lambda)\delta(s_H\bar{\theta}-2p_H^2+c_H)=0,\quad (A18)$$
$$\Pi_H-p_H^1Q_H^1\ge 0;\lambda\ge 0;\lambda(\Pi_H-p_H^1Q_H^1)=0.\quad (A19)$$

Solving (A17) and (A18), we obtain

$$p_H^1=\frac{s_H\bar{\theta}}{2}+\frac{c_H}{2}+\frac{\lambda c_H}{2};\ p_H^2=\frac{s_H\bar{\theta}}{2}+\frac{c_H}{2},\quad (A20)$$

Using the prices in (A20), we rewrite the IC constraint (23) as

$$\delta s_H^2\bar{\theta}^2-2(1+\delta)s_H c_H\bar{\theta}+(2+\delta+2\lambda)c_H^2\ge 0.\quad (A21)$$

In the case where the IC constraint is slack, $\lambda=0$ and both prices in (A20) are equal to p_H^*; in other words, the equilibrium is identical to that under perfect information. In this case, (A21) (with $\lambda=0$) becomes $\delta s_H^2\bar{\theta}^2-2(1+\delta)s_H c_H\bar{\theta}+(2+\delta)c_H^2\ge 0$, which holds if $c_H\le c_H^{\gamma}$. This proves part a) of Lemma 2.

In the case where $\lambda>0$, the IC constraint is binding and thus (A21) holds with equality. Solving (A21) for λ, we obtain

$$\lambda=\frac{-[\delta s_H\bar{\theta}-(2+\delta)c_H](s_H\bar{\theta}-c_H)}{2c_H^2}.\quad (A22)$$

Since $c_H\le s_H\bar{\theta}(\equiv c_H^{\alpha})$, (A22) implies that $c_H>c_H^{\gamma}$ is needed to satisfy $\lambda>0$. Moreover, for the quantity of the PL in period 1 to be positive, we need

$$\hat{Q}_H^1=\frac{\delta(s_H\bar{\theta}-c_H)^2}{4s_H c_H}>0,\quad (A23)$$

which always holds. Therefore, in the case where $c_H^{\gamma}<c_H<c_H^{\alpha}$, we have $p_H^1=\hat{p}_H, p_H^2=p_H^*$, $Q_H^1=\hat{Q}_H$, and $Q_H^2=Q_H^*$. Using the equilibrium prices and quantities, we find $\Pi_H=\widehat{\Pi}_H$. This proves part b) of Lemma 2. Q.E.D.

Lemma A and its Proof:

Lemma A. Under asymmetric information, the prices and quantities sold in the equilibrium of the continuation game associated with the product line (NB, PL) depend on the values of c_H and c_B. If $c_H \leq c_H^{\gamma}$, the equilibrium prices and quantities of the two products are the same as those under perfect information for c_B in the relevant intervals. If $c_H^{\gamma} < c_H < c_H^{\alpha}$, on the other hand, the equilibrium prices and quantities depend on the value of c_B as follows.

a) In the case where $c_B \in (c_H, c_B^c]$, $Q_H^1 = Q_H^2 = 0$ and $Q_B^1 = Q_B^2 > 0$; that is, the quantity of the private label product sold is 0 while the quantity of the national brand product sold is positive in both periods. Moreover, the equilibrium prices and quantities of the national brand product in both periods are the same as those under perfect information.

b) In the case where $c_B \in \left(c_B^c, c_B^b\right)$, $Q_H^1 > 0$, $Q_B^1 > 0$, $Q_H^2 > 0$, and $Q_B^2 > 0$; that is, the quantities sold are positive for both the private label and national brand product in both periods. Moreover, the equilibrium prices and quantities of the two products in both periods are the same as those under perfect information if c_B is in the range $\left(c_B^c, c_B^{c'}\right)$.

c) In the case where $c_B \in [c_B^b, c_B^{b'})$, $Q_H^1 > 0$, $Q_B^1 > 0$, $Q_H^2 > 0$, but $Q_B^2 = 0$; that is, the quantity of the private label product sold is positive in both periods, but the quantity of the national brand product sold is positive in period 1 only. Moreover, the equilibrium price of the private label product and quantities of the two products in period 1 are different from those under perfect information.

d) In the case where $c_B \in [c_B^{b'}, c_B^{\alpha})$, $Q_H^1 > 0$, $Q_H^2 > 0$ and $Q_B^1 = Q_B^2 = 0$; that is, the quantity of the private label product sold is positive while the quantity of the national brand product sold is 0 in both periods. Moreover, the equilibrium price and quantity of the private label product in period 1 are different from those under perfect information.

Proof: The prices and quantities sold in the equilibrium of the continuation game associated with the product line (NB, PL) are solved from the retailer's optimization problem (27)-(28). The Lagrange function associated with this problem is

$$L(p_B^1, p_H^1, p_B^2, p_H^2, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5) = \pi_{BH} + \mu_1(\pi_{BH} - \pi_{BL}) + \mu_2 Q_B^1 + \mu_3 Q_H^1 + \mu_4 Q_B^2 + \mu_5 Q_H^2. \quad (A24)$$

Solving the first-order conditions with respect to p_B^1, p_H^1, p_B^2 and p_H^2 yields

$$p_B^1 = \frac{s_B\bar{\theta}}{2} + \frac{c_B}{2} - \frac{\mu_2}{2}, \qquad p_H^1 = \frac{s_H\bar{\theta}}{2} + \frac{c_H}{2} + \frac{\mu_1 c_H}{2} - \frac{\mu_3}{2}, \qquad (A25)$$

$$p_B^2 = \frac{s_B\bar{\theta}}{2} + \frac{c_B}{2} - \frac{\mu_4}{2\delta(1+\mu_1)}, \qquad p_H^2 = \frac{s_H\bar{\theta}}{2} + \frac{c_H}{2} - \frac{\mu_5}{2\delta(1+\mu_1)}. \qquad (A26)$$

Substituting the prices in (A25) and (A26) into the demand functions, we have

$$Q_B^1 = \frac{\bar{\theta}}{2} - \frac{c_B - c_H - \mu_1 c_H - \mu_2 + \mu_3}{2(s_B - s_H)}, \qquad (A27)$$

$$Q_H^1 = \frac{s_H c_B - s_B c_H - \mu_1 s_B c_H - \mu_2 s_H + \mu_3 s_B}{2 s_H (s_B - s_H)}, \qquad (A28)$$

$$Q_B^2 = \frac{\bar{\theta}}{2} - \frac{c_B - c_H - \frac{\mu_4}{\delta(1+\mu_1)} + \frac{\mu_5}{\delta(1+\mu_1)}}{2(s_B - s_H)}, \qquad (A29)$$

$$Q_H^2 = \frac{s_H c_B - s_B c_H - \frac{\mu_4 s_H}{\delta(1+\mu_1)} + \frac{\mu_5 s_B}{\delta(1+\mu_1)}}{2 s_H (s_B - s_H)}. \qquad (A30)$$

The IC constraint becomes

$$\pi_{BH} - \pi_{BL} = -\frac{c_H (s_H c_B - s_B c_H - \mu_1 s_B c_H - \mu_2 s_H + \mu_3 s_B)}{2 s_H (s_B - s_H)}$$
$$+ \frac{\delta \left[s_B \bar{\theta} - c_B - \frac{\mu_4}{\delta(1+\mu_1)} \right] \left[\bar{\theta}(s_B - s_H) - c_B + c_H + \frac{\mu_4}{\delta(1+\mu_1)} - \frac{\mu_5}{\delta(1+\mu_1)} \right]}{4(s_B - s_H)}$$
$$+ \frac{\delta \left[s_H \bar{\theta} - c_H - \frac{\mu_5}{\delta(1+\mu_1)} \right] \left[s_H c_B - s_B c_H - \frac{\mu_4 s_H}{\delta(1+\mu_1)} + \frac{\mu_5 s_B}{\delta(1+\mu_1)} \right]}{4 s_H (s_B - s_H)} \geq 0. \qquad (A31)$$

Lemma A lists four different cases in terms of the quantities of the two products in each of the two periods. In the ensuing proof, we use (A27)-(A31) to find out the ranges of c_H and c_B for which each of the four cases would arise. We will also note the conditions under which the equilibrium quantities and prices are the same as those under perfect information.

In the case where the quantity of the NB product is positive and the quantity of the PL product is zero in both periods, $\mu_2 = \mu_4 = 0$; $\mu_3 \geq 0$, $\mu_5 \geq 0$ and solving (A28) and (A30) we have

$$\mu_1 c_H - \mu_3 = \frac{s_H c_B - s_B c_H}{s_B}, \qquad (A32)$$

$$\frac{\mu_5}{\delta(1+\mu_1)} = -\frac{s_H c_B - s_B c_H}{s_B}. \qquad (A33)$$

Since $\mu_1 \geq 0$ and $\mu_5 \geq 0$, (A33) implies that $c_B \leq c_B^c$. If the IC constraint is slack and hence $\mu_1 = 0$, (A32) also entails $c_B \leq c_B^c$. Moreover, (A27) and (A29) imply that $c_B < c_B^a$. In this case, (A31) becomes $[s_H(s_B\bar{\theta} - c_B)^2(s_B - s_H)]/s_B > 0$ which is satisfied because $s_B > s_H$ and $c_B < c_B^a$. If the IC constraint is binding, $\mu_1 > 0$. Substituting (A32) and (A33) into (A27) and (A29), respectively, we have $c_B < c_B^a$. On the other hand, the binding (A31) entails $c_B = s_B\bar{\theta} (\equiv c_B^a)$, which contradicts the condition $c_B < c_B^a$. This implies that the IC constraint is always slack in the case where the quantity of the NB product is positive and the quantity of the PL product is zero in both periods. Therefore, in the case where $c_B \in (c_H, c_B^c]$, we have $p_B^1 = p_B^2 = p_B^*$ and the prices of the PL are given in (A25) and (A26), $Q_B^1 = Q_B^2 = Q_B^*$ and $Q_H^1 = Q_H^2 = 0$. Using the equilibrium prices and quantities, we find the equilibrium joint profit in this case is Π_B^*. This proves part a) of Lemma A.

In part b) of Lemma A, the quantities of both products are positive in both periods, in which case $\mu_2 = \mu_3 = \mu_4 = \mu_5 = 0$. Solving (A27)-(A30) yields

$$Q_B^1 = \frac{\bar{\theta}}{2} - \frac{c_B - c_H - \mu_1 c_H}{2(s_B - s_H)} > 0, \qquad (A34)$$

$$Q_H^1 = \frac{s_H c_B - s_B c_H - \mu_1 s_B c_H}{2 s_H (s_B - s_H)} > 0, \qquad (A35)$$

$$Q_B^2 = \frac{\bar{\theta}}{2} - \frac{c_B - c_H}{2(s_B - s_H)} > 0, \qquad (A36)$$

$$Q_H^2 = \frac{s_H c_B - s_B c_H}{2s_H(s_B - s_H)} > 0. \qquad (A37)$$

(A36) and (A37) imply that $c_B \in \left(c_B^c, c_B^b\right)$.

If the IC constraint is slack and hence $\mu_1 = 0$, (A34) and (A35) also imply that $c_B \in \left(c_B^c, c_B^b\right)$. In this case, (A31) becomes $\delta s_H c_B^2 - 2s_H c_B[\delta\bar{\theta}(s_B - s_H) + (1+\delta)c_H] + s_B[(2+\delta)c_H^2 + \delta s_H \bar{\theta}^2(s_B - s_H)] > 0$. This inequality holds for $c_B \in \left(c_B^c, c_B^b\right)$ when $c_H \le c_H^\gamma$ and for $c_B \in \left(c_B^c, c_B^{c'}\right)$ when $c_H^\gamma < c_H < c_H^\alpha$, where $c_B^{c'}$ is defined in (30). Note that when the IC constraint is slack, the quantities of both products in both periods are the same as those under perfect information. The above analysis shows that this is true under each of the following circumstances: (i) $c_H \le c_H^\gamma$ and $c_B \in \left(c_B^c, c_B^b\right)$; (ii) $c_H^\gamma < c_H < c_H^\alpha$, and $c_B \in \left(c_B^c, c_B^{c'}\right)$.

If the IC constraint is binding, $\mu_1 > 0$ and (A31) holds with equality. Solving (A31) we obtain

$$\mu_1 = \frac{(s_H c_B - s_B c_H)[(2+\delta)c_H - \delta s_H \bar{\theta}] - \delta s_H (s_B \bar{\theta} - c_B)[\bar{\theta}(s_B - s_H) - c_B + c_H]}{2s_B c_H^2}. \qquad (A38)$$

(A34) and (A35) imply that

$$\frac{-\bar{\theta}(s_B - s_H) + c_B - c_H}{c_H} < \mu_1 < \frac{s_H c_B - s_B c_H}{s_B c_H}. \qquad (A39)$$

Conditions (A38)-(A39), along with $\mu_1 > 0$, entail $c_H^\gamma < c_H < c_H^\alpha$ and $c_B^{c'} < c_B < c_B^b$. Within these ranges of parameter values, the quantities of both products are positive in both periods. Because of the binding IC constraint, however, we have $Q_B^1 > Q_{B|BH}^*$, $Q_H^1 < Q_{H|BH}^*$. For later analysis, note that the equilibrium joint profit in this case is $\widehat{\Pi}_{BH} = \Pi_{BH}^* - \mu_1^2 s_B c_H^2 / 4s_H(s_B - s_H)$.

In the case where the quantity of the NB is positive in period 1 and zero in period 2 and the quantity of the PL is positive in both periods, $\mu_2 = \mu_3 = \mu_5 = 0$ and $\mu_4 \ge 0$. Substituting these values into (A27)-(A30), we find

$$Q_B^1 = \frac{\bar{\theta}}{2} - \frac{c_B - c_H - \mu_1 c_H}{2(s_B - s_H)} > 0, \qquad (A40)$$

$$Q_H^1 = \frac{s_H c_B - s_B c_H - \mu_1 s_B c_H}{2s_H(s_B - s_H)} > 0, \qquad (A41)$$

$$Q_B^2 = \frac{\bar{\theta}}{2} - \frac{c_B - c_H - \frac{\mu_4}{\delta(1+\mu_1)}}{2(s_B - s_H)} = 0, \qquad (A42)$$

$$Q_H^2 = \frac{s_H c_B - s_B c_H - \frac{\mu_4 s_H}{\delta(1+\mu_1)}}{2s_H(s_B - s_H)} > 0. \qquad (A43)$$

(A42) and (A43) imply that $c_H < c_H^\alpha$. Suppose the IC constraint is slack. Setting $\mu_1 = 0$ in (A40), we find $c_B < c_B^b$. Solving (A42), we obtain $\mu_4 = -\delta[\bar{\theta}(s_B - s_H) - c_B + c_H]$, which has a negative sign if $c_B < c_B^b$. Therefore, a slack IC constraint is not possible in this case. Now suppose the IC constraint is binding, in which case $\mu_1 > 0$ and (A31) holds with equality. Solving (A31), we obtain

$$\mu_1 = \frac{2c_H(s_H c_B - s_B c_H) - \delta(s_B - s_H)(s_H \bar{\theta} - c_H)^2}{2s_B c_H^2}. \qquad (A44)$$

From (A44), we see that $\mu_1 > 0$ implies $c_B > [\delta(s_B - s_H)(s_H\bar{\theta} - c_H)^2 + 2s_B c_H^2]/2s_H c_H$, which holds for $c_H > c_H^\gamma$. Substituting (A44) into (A40), we find that $c_B < c_B^{b'}$ for $c_H \in \left(c_H^\gamma, c_H^\alpha\right)$, where $c_B^{b'}$ is defined in (29). Substituting (A44) into (A41), we obtain $-\delta(s_B - s_H)(s_H\bar{\theta} - c_H)^2 < 0$, which always holds. Moreover, Using (A44) to solve (A42), we find $\mu_4 = \delta(1+\mu_1)[-\bar{\theta}(s_B - s_H) + c_B - c_H]$, and $\mu_4 \geq 0$ implies that $c_B \geq c_B^b$. Therefore, when $c_H \in \left(c_H^\gamma, c_H^\alpha\right)$ and $c_B \in [c_B^b, c_B^{b'})$, $Q_B^1 = Q_{B|BH}^* + \mu_1[c_H/2(s_B - s_H)] > 0$ and $Q_H^1 = Q_{H|BH}^* - \mu_1 s_B[c_H/2s_H(s_B - s_H)] > 0$, $Q_B^2 = 0$ and $Q_H^2 = Q_H^*$. This proves part c) of Lemma A. For later analysis, note that the equilibrium joint profit in this case is equal to $(\Pi_{BH}^* + \delta\Pi_H^*)/(1+\delta) - s_B\mu_1^2 c_H^2/4s_H(s_B - s_H)$.

In the case where the quantity of the NB product is zero and the quantity of the PL product is positive in both periods, $\mu_3 = \mu_5 = 0$ and $\mu_2 \geq 0$, $\mu_4 \geq 0$. Substituting these values into (A27)-(A30) to find

$$Q_B^1 = \frac{\bar{\theta}}{2} - \frac{c_B - c_H - \mu_1 c_H - \mu_2}{2(s_B - s_H)} = 0, \qquad (A45)$$

$$Q_H^1 = \frac{s_H c_B - s_B c_H - \mu_1 s_B c_H - \mu_2 s_H}{2s_H(s_B - s_H)} > 0, \qquad (A46)$$

$$Q_B^2 = \frac{\bar{\theta}}{2} - \frac{c_B - c_H - \dfrac{\mu_4}{\delta(1+\mu_1)}}{2(s_B - s_H)} = 0, \qquad (A47)$$

$$Q_H^2 = \frac{s_H c_B - s_B c_H - \dfrac{\mu_4 s_H}{\delta(1+\mu_1)}}{2s_H(s_B - s_H)} > 0. \qquad (A48)$$

Suppose the IC constraint is slack and hence $\mu_1 = 0$. Solving for μ_2 from (A45) and μ_4 from (A47), we find that both $\mu_2 \geq 0$ and $\mu_4 \geq 0$ imply $c_B \geq c_B^b$. From (A46) and (A48) we conclude that $c_H < c_H^\alpha$. Moreover, (A31) becomes $(s_H\bar{\theta} - c_H)[-2c_H + \delta(s_H\bar{\theta} - c_H)] \geq 0$, which implies that $c_H \leq c_H^\gamma$. Therefore, $Q_B^1 = Q_B^2 = 0$ and $Q_H^1 = Q_H^2 = Q_H^*$ if $c_H \leq c_H^\gamma$ and $c_B \geq c_B^b$.

Now suppose the IC constraint is binding. Then (A31) holds with equality and $\mu_1 > 0$. Solving (A31) to find that

$$\mu_1 = \frac{-\delta(s_H\bar{\theta} - c_H)^2 + 2c_H(s_H\bar{\theta} - c_H)}{2c_H^2} > 0 \qquad (A49)$$

implies $c_H > c_H^\gamma$. From (A45) we obtain $\mu_2 = -\bar{\theta}(s_B - s_H) + c_B - c_H - \mu_1 c_H$, and $\mu_2 \geq 0$ implies that $c_B \geq c_B^{b'}$. Substituting the preceding expression of μ_2 into (A46), we obtain $\mu_1 < (s_H\bar{\theta} - c_H)/c_H$. The latter is satisfied by (A49). Solving $\mu_4/(1+\mu_1)$ from (A47) substituting it into (A48), we find $Q_H^2 = Q_H^*$. Therefore, when $c_H \in \left(c_H^\gamma, c_H^\alpha\right)$ and $c_B \in [c_B^{b'}, c_B^a)$, $Q_B^1 = Q_B^2 = 0$, $Q_H^1 = Q_H^* - \mu_1 c_H/2s_H > 0$ and $Q_H^2 = Q_H^*$. This proves part d) of Lemma A. Q.E.D.

Proof of Proposition 4: We determine the equilibrium product line by comparing the retailer's profits associated with the three product lines: (NB, 0), (NB, PL) and (0, PL). The retailer's profit associated with (NB, 0) is $\omega\Pi_B^*$, where Π_B^* is given in (15). Its profit associated with (0, PL) is $\widehat{\Pi}_H - k$, where $\widehat{\Pi}_H$ can be found in Lemma 2b). As indicated in Lemma A, the retailer's profit associated with (NB, PL) depends on the value of c_B.

First, consider the case where $c_B \in (c_H, c_B^c]$. By part a) of Lemma A, the quantity of the PL product sold is zero in both periods if the retailer chooses the product line (NB, PL). Accordingly, the retailer's profit associated with this product line is $\omega\Pi_B^* + (1-\omega)\widehat{\Pi}_H - k$. This product line is more profitable than (NB, 0) because $(1-\omega)\widehat{\Pi}_H - k > 0$ for a small k. Moreover, (NB, PL) is more profitable than (0, PL) if $\omega(\Pi_B^* -$

$\Pi_H^*) - k > 0$. As noted in the proof of Proposition 1, $\Pi_B^* > \Pi_H^*$ for $c_B \in (c_H, c_B^c]$. Therefore, the retailer earns the largest profit from (NB, PL) in this case, and the quantity of the PL sold is zero in both periods.

In the case of $c_B \in \left(c_B^c, c_B^b\right)$, we need to consider the following two sub-cases: (i) $c_B \in \left(c_B^c, c_B^{c'}\right)$, and (ii) $c_B \in \left(c_B^{c'}, c_B^b\right)$. As shown in the proof of Lemma A, if the retailer chooses the product line (NB, PL) in the first sub-case, the quantities sold are equal to those under perfect information. Accordingly, its profit associated with this product line is $\omega\Pi_{BH}^* + (1-\omega)\widehat{\Pi}_H - k$. Recall that the retailer's profits associated with (NB, 0) and (0, PL) are $\omega\Pi_B^*$ and $\widehat{\Pi}_H - k$, respectively. As shown in the proof of Proposition 1, $\Pi_{BH}^* > \Pi_B^*$ and $\Pi_{BH}^* > \Pi_H^*$ for $c_B \in \left(c_B^c, c_B^b\right)$. Combining these results with the observation $\Pi_H^* > \widehat{\Pi}_H$, we conclude that the retailer earns the largest profit from (NB, PL) in this case and the quantities of both products are positive in both periods.

In the second sub-case, the quantities associated with product line (NB, PL) are not the same as those under perfect information. Accordingly, the retailer's profit associated with this product line is $\omega\widehat{\Pi}_{BH} + (1-\omega)\widehat{\Pi}_H - k$. The retailer's profits associated with (NB, 0) and (0, PL) are the same as those specified above. Hence, (NB, PL) is more profitable than (0, PL) if $\widehat{\Pi}_{BH} > \widehat{\Pi}_H$. Using the results from the proofs of Lemma 1 and Lemma A, we can show that,

$$\widehat{\Pi}_{BH} - \widehat{\Pi}_H = \Pi_{BH}^* - \Pi_H^* + F(c_B) \text{ with } F(c_B) = \frac{\lambda(c_B)^2 c_H^2}{4s_H} - \frac{s_B\mu_1(c_B)^2 c_H^2}{4s_H(s_B - s_H)}. \qquad (A50)$$

It can be shown that $F'(c_B) < 0$ for $c_B \in \left(c_B^c, c_B^b\right)$, and $\left(c_B^b\right) = [(s_H\bar{\theta} - c_H)/c_H]^2(2+\delta)^2 \left[\left(c_H - c_H^\gamma\right)^2/16s_B\right] > 0$. They imply that $F(c_B) > 0$ for $c_B \in \left(c_B^{c'}, c_B^b\right)$. From Proposition 1, we know that $\Pi_{BH}^* - \Pi_H^* > 0$ for $c_B \in \left(c_B^{c'}, c_B^b\right)$. Then from (A50) we conclude that $\widehat{\Pi}_{BH} - \widehat{\Pi}_H > 0$ for $c_B \in (c_B^{c'}, c_B^b)$. This proves that (NB, PL) is more profitable than (0, PL) in this sub-case. On the other hand, (NB, PL) is more profitable than (NB, 0) if $\omega\left(\widehat{\Pi}_{BH} - \Pi_B^*\right) + (1-\omega)\widehat{\Pi}_H - k > 0$. It can be shown that $\partial\left(\widehat{\Pi}_{BH} - \Pi_B^*\right)/\partial c_B = -\delta s_H\left(c_B^b - c_B\right)/c_H < 0$ for $c_B \in \left(c_B^c, c_B^b\right)$. Moreover, when $c_B = c_B^b$, $\widehat{\Pi}_{BH} - \Pi_B^* = \bar{\theta}(4+\delta)[1 - \delta s_H/(4+\delta)][(s_H\bar{\theta} - c_H)^3/16s_Bs_Hc_H^2] > 0$. Thus, we can infer that $\widehat{\Pi}_{BH} - \Pi_B^* > 0$ for $c_B \in \left(c_B^{c'}, c_B^b\right)$. This, in turn, implies that (NB, PL) is more profitable than (NB, 0) for a small k. Therefore, the retailer earns the largest profit from (NB, PL) when $c_B \in \left(c_B^{c'}, c_B^b\right)$.

In the case where $c_B \in \left[c_B^b, c_B^{b'}\right)$, the retailer's profits associated with (NB, 0), (NB, PL) and (0, PL) are $\omega\Pi_B^*$, $\omega\widehat{\Pi}_{BH} + (1-\omega)\widehat{\Pi}_H - k$ and $\widehat{\Pi}_H - k$, respectively. Moreover, $\widehat{\Pi}_{BH}$ in this case is $(\Pi_{BH}^* + \delta\Pi_H^*)/(1+\delta) - s_Bc_H^2\mu_1^2/4s_H(s_B - s_H)$, where the expression of μ_1 is provided in the proof of Lemma A. (NB, PL) is more profitable than (0, PL) if $(\Pi_{BH}^* - \Pi_H^*)/(1+\delta) + \lambda^2c_H^2/4s_H - s_B\mu_1^2c_H^2/4s_H(s_B - s_H) > 0$. As shown in the proof of Proposition 1, $\Pi_{BH}^* - \Pi_H^* > 0$. Furthermore, it can be shown that $\lambda^2c_H^2/4s_H - s_B\mu_1^2c_H^2/4s_H(s_B - s_H)$ is decreasing in $c_B \in \left[c_B^b, c_B^{b'}\right)$, and at $c_B = c_B^{b'}$, $\lambda^2c_H^2/4s_H - s_B\mu_1^2c_H^2/4s_H(s_B - s_H) = 0$. They imply that $\lambda^2c_H^2/4s_H - s_B\mu_1^2c_H^2/4s_H(s_B - s_H) > 0$ for $c_B \in \left[c_B^b, c_B^{b'}\right)$. Hence, (NB, PL) is more profitable than (0, PL) for $c_B \in \left[c_B^b, c_B^{b'}\right)$. Note that (NB, PL) is more profitable than (NB, 0) if $\omega[(\Pi_{BH}^* - \Pi_B^*)/(1+\delta) - s_B\mu_1^2c_H^2/4s_H(s_B - s_H) + (\Pi_H^* - \Pi_B^*)/(1+\delta)] + (1-\omega)\widehat{\Pi}_H - k > 0$. It can be shown that $\partial[(\Pi_{BH}^* - \Pi_B^*)/(1+\delta) - s_B\mu_1^2c_H^2/4s_H(s_B - s_H)]/\partial c_B = -\delta s_H[(s_B - s_H)\bar{\theta} - c_B + c_H]/c_H > 0$ for $c_B \in \left(c_B^b, c_B^{b'}\right)$. At $c_B = c_B^b$, $(\Pi_{BH}^* - \Pi_B^*)/(1+\delta) - s_B\mu_1^2c_H^2/4s_H(s_B - s_H) = (4+\delta)\,\bar{\theta}(s_H\bar{\theta} - c_H)^3[1 - \delta s_H/(4+\delta)]/16s_Bs_Hc_H^2 > 0$. Thus, for $c_B \in \left(c_B^b, c_B^{b'}\right)$, $(\Pi_{BH}^* - \Pi_B^*)/(1+\delta) - s_B\mu_1^2c_H^2/4s_H(s_B - s_H) > 0$. Moreover, as shown in the proof of Proposition 1, $\Pi_H^* - \Pi_B^* > 0$. These observations imply that $\widehat{\Pi}_{BH} > \Pi_B^*$. Since $(1-\omega)\widehat{\Pi}_H - k > 0$ for a small k, we conclude that (NB, PL) is more profitable than (NB, 0). Therefore, the retailer earns the largest profit from (NB, PL) when $c_B \in \left[c_B^b, c_B^{b'}\right)$.

In the case where $c_B \in \left[c_B^{b'}, c_B^a\right)$, the retailer would sell 0 units of the NB product even if chooses the product line (NB, PL). Consequently, its profit associated with (NB, PL) is $\widehat{\Pi}_H - k$, the same as that with (0, PL). Since the retailer does not gain any additional profit from carrying the NB product, the retailer chooses (0, PL) over (NB, PL). In comparison with the profit associated with (0, NB), note that $\widehat{\Pi}_H - \Pi_B^*$ is increasing in $c_B \in \left[c_B^{b'}, c_B^a\right)$, and at $c_B = c_B^{b'}$, $\widehat{\Pi}_H - \Pi_B^* = \delta(s_H\bar{\theta} - c_H)^2\ [2s_Bc_H - \delta(s_B + s_H)(s_H\bar{\theta} - c_H)]/16s_Bs_Hc_H^2 > 0$. These imply that (0, PL) is more profitable than (NB, 0) for $c_B \in \left[c_B^{b'}, c_B^a\right)$. Therefore, the retailer earns the largest profit from (0, PL) when $c_B \in \left[c_B^{b'}, c_B^a\right)$. Q.E.D.

Proof of Proposition 5: Since the quantities sold in the second period under asymmetric information are the same as those under perfect information, we need to analyze the difference in quantities in the first period only.

Consider the quantity of the PL product. When $c_B \in (c_B^{c'}, c_B^b)$, the first period equilibrium quantity of the PL product is represented by (A35), which can be rewritten as:

$$Q_H^1 = Q_{H|BH}^* - \frac{(2+\delta)(c_B - c_B^c)\left(c_H - c_H^\gamma\right) - \delta(c_B^a - c_B)\left(c_B^b - c_B\right)}{4(s_B - s_H)c_H}. \qquad (A51)$$

From (A51), we find that $Q_H^1 < Q_{H|BH}^*$ for $c_H \in \left(c_H^\gamma, c_H^\alpha\right)$ and $c_B \in \left(c_B^{c'}, c_B^b\right)$. When $c_B \in \left(c_B^b, c_B^{b'}\right)$, the first period equilibrium quantity of the PL product is given by (A41), which can be rewritten as

$$Q_H^1 = Q_{H|BH}^* - \frac{2s_Hc_H(c_B - c_B^c) - \delta(s_B - s_H)(c_H^a - c_H)^2}{4s_H(s_B - s_H)c_H}. \qquad (A52)$$

Using (A52), we can show that $Q_H^1 < Q_{H|BH}^*$ for $c_H \in \left(c_H^\gamma, c_H^\alpha\right)$ and $c_B \in \left(c_B^b, c_B^{b'}\right)$. From the last paragraph in the proof of Lemma A, we see that when $c_B \in \left(c_B^{b'}, c_B^a\right)$, the equilibrium quantity of the PL product in period 1 is

$$Q_H^1 = Q_H^* - \frac{(2+\delta)(c_H^\alpha - c_H)\left(c_H^\gamma - c_H\right)}{4s_Hc_H}, \qquad (A53)$$

which is smaller than Q_H^* for $c_H \in \left(c_H^\gamma, c_H^\alpha\right)$. Therefore, the quantity of the PL product sold under asymmetric information is smaller than that under perfect information if $c_B \in \left(c_B^{c'}, c_B^a\right)$. As noted in the proof of Proposition 4, when $c_B \in \left(c_H, c_B^{c'}\right)$ the quantity of the PL product sold under asymmetric information is the same as that under perfect information.

Next, consider the quantity of the NB product. In the case of $c_B \in \left(c_B^{c'}, c_B^b\right)$, the first period equilibrium quantity of the NB is given by (A34), which can be rewritten as

$$Q_B^1 = Q_{B|BH}^* + \frac{s_H\left[(2+\delta)(c_B - c_B^c)\left(c_H - c_H^\gamma\right) - \delta(c_B^a - c_B)\left(c_B^b - c_B\right)\right]}{4s_B(s_B - s_H)c_H}. \qquad (A54)$$

From (A54) we find that $Q_B^1 > Q_{B|BH}^*$ for $c_H \in \left(c_H^\gamma, c_H^\alpha\right)$ and $c_B \in \left(c_B^{c'}, c_B^b\right)$. When $c_B \in \left(c_B^b, c_B^{b'}\right)$, the equilibrium quantity of the NB product in period 1 is represented by (A40), which can be rewritten as

$$Q_B^1 = \frac{2s_Hc_H(c_B - c_B^c) - \delta(s_B - s_H)(c_H^\alpha - c_H)^2}{4s_B(s_B - s_H)c_H}. \qquad (A55)$$

From (A55) we find that $Q_B^1 > 0$ for $c_H \in \left(c_H^\gamma, c_H^\alpha\right)$ and $c_B \in \left(c_B^b, c_B^{b'}\right)$. Recalling that for this range of parameter values the quantity of the NB product is 0 under perfect information, we conclude that $Q_B^1 > Q_B^*$ in this case. Therefore, the quantity of the NB sold under asymmetric information is larger than that under perfect information if $c_B \in (c_B^{c'},\ c_B^{b'})$. As noted in the proofs of Lemma A and Proposition 4, the quantity of the NB product sold under asymmetric information is the same as that under perfect information if $c_B \in (c_H, c_B^{c'})$ and if $c_B \in (c_B^{b'}, c_B^{a})$. Q.E.D.

Proof of Proposition 6: For ease of comparison, in Table A1 we present the retailer's profits under perfect and asymmetric information for $c_H \in (c_H^{\gamma}, c_H^{\alpha})$ and c_B in different intervals. Since $\widehat{\Pi}_H < \Pi_H^*$ and $\widehat{\Pi}_{BH} < \Pi_{BH}^*$ for $c_H \in (c_H^{\gamma}, c_H^{\alpha})$, it is straightforward to show that the retailer earns a smaller profit under asymmetric information than under perfect information for c_B in the intervals $(c_H, c_B^c]$, $(c_B^c, c_B^{c'}]$, $(c_B^{c'}, c_B^b)$ and $[c_B^{b'}, c_B^a)$. When $c_B \in [c_B^b, c_B^{b'})$, the retailer earns a larger profit under perfect information if $\Pi_H^* > \omega\widehat{\Pi}_{BH} + (1-\omega)\widehat{\Pi}_H$. Recall from the proof of Proposition 4 that $\widehat{\Pi}_{BH} = (\Pi_{BH}^* + \delta\Pi_H^*)/(1+\delta) - s_B c_H^2 \mu_1^2/4s_H(s_B - s_H)$, which is less than Π_H^* because $\Pi_{BH}^* < \Pi_H^*$ for $c_B \in [c_B^b, c_B^a)$. This, in conjunction with $\Pi_H^* > \widehat{\Pi}_H$, implies that the retailer's profit is larger under perfect information than under asymmetric information when $c_B \in [c_B^b, c_B^{b'})$.

Table A1 The Retailer's Profits

$\boldsymbol{c_B}$	Profits under perfect information	Profits under asymmetric information
$(\boldsymbol{c_H}, \boldsymbol{c_B^c}]$	$\omega\Pi_B^* + (1-\omega)\Pi_H^* - k$	$\omega\Pi_B^* + (1-\omega)\widehat{\Pi}_H - k$
$(\boldsymbol{c_B^c}, \boldsymbol{c_B^{c'}}]$	$\omega\Pi_{BH}^* + (1-\omega)\Pi_H^* - k$	$\omega\Pi_{BH}^* + (1-\omega)\widehat{\Pi}_H - k$
$(\boldsymbol{c_B^{c'}}, \boldsymbol{c_B^b})$	$\omega\Pi_{BH}^* + (1-\omega)\Pi_H^* - k$	$\omega\widehat{\Pi}_{BH} + (1-\omega)\widehat{\Pi}_H - k$
$[\boldsymbol{c_B^b}, \boldsymbol{c_B^{b'}})$	$\Pi_H^* - k$	$\omega\widehat{\Pi}_{BH} + (1-\omega)\widehat{\Pi}_H - k$
$[\boldsymbol{c_B^{b'}}, \boldsymbol{c_B^a})$	$\Pi_H^* - k$	$\widehat{\Pi}_H - k$

Table A2 The NB Supplier's Profits

$\boldsymbol{c_B}$	Profits under perfect information	Profits under asymmetric information
$(\boldsymbol{c_H}, \boldsymbol{c_B^c}]$	$(1-\omega)\Pi_B^* - (1-\omega)\Pi_H^*$	$(1-\omega)\Pi_B^* - (1-\omega)\widehat{\Pi}_H$
$(\boldsymbol{c_B^c}, \boldsymbol{c_B^{c'}}]$	$(1-\omega)\Pi_{BH}^* - (1-\omega)\Pi_H^*$	$(1-\omega)\Pi_{BH}^* - (1-\omega)\widehat{\Pi}_H$
$(\boldsymbol{c_B^{c'}}, \boldsymbol{c_B^b})$	$(1-\omega)\Pi_{BH}^* - (1-\omega)\Pi_H^*$	$(1-\omega)\widehat{\Pi}_{BH} - (1-\omega)\widehat{\Pi}_H$
$[\boldsymbol{c_B^b}, \boldsymbol{c_B^{b'}})$	0	$(1-\omega)\widehat{\Pi}_{BH} - (1-\omega)\widehat{\Pi}_H$

In Table A2 are the NB supplier's profits for $c_H \in (c_H^{\gamma}, c_H^{\alpha})$ and c_B in different intervals. Using the same logic as above, we can show that the NB supplier earns a larger profit under asymmetric information than under perfect information for c_B in the intervals $(c_H, c_B^c]$ and $(c_B^c, c_B^{c'}]$. When $c_B \in (c_B^{c'}, c_B^b)$, the NB supplier earns larger profit under asymmetric information if $\lambda^2 c_H^2/4s_H - \widehat{\Pi}_{BH} - \widehat{\Pi}_H - (\Pi_{BH}^* - \Pi_H^*) > 0$. By (A50), this inequality is equivalent to $s_B\mu_1^2 c_H^2/4s_H(s_B - s_H) > 0$, which is shown to be true in the proof of Proposition 4. Thus the supplier's profit is larger under asymmetric information than under perfect information when $c_B \in (c_B^{c'}, c_B^b)$. When $c_B \in [c_B^b, c_B^{b'})$, the NB supplier earns a larger profit under asymmetric information than under perfect information because, by Proposition 4, $\widehat{\Pi}_{BH} - \widehat{\Pi}_H > 0$. Q.E.D.

Proof of Proposition 7: As shown in the proof of Proposition 4, the equilibrium quantities under asymmetric information are the same as those under perfect information if $c_B \in (c_B^c, c_B^{c'})$. In this case, the combined quantity of the two products in each period associated with (NB, PL) is equal to $Q^*_{B|BH} + Q^*_{H|BH}$ given in (A14). The total quantity associated with (NB, 0), on the other hand, is equal to Q^*_B in (A15). Since $c_B > c_B^c$, from (A14) and (A15) we find that $Q^*_{B|BH} + Q^*_{H|BH} > Q^*_B$; that is, the combined quantity of (NB, PL) is larger than the quantity of (NB, 0). The joint profit associated with (NB, PL) and (NB, 0) are Π^*_{BH} and Π^*_B, respectively. Eq. (A10) shows that the former is larger than the latter.

For the remaining cases, we need to consider only the quantities in period 1 because the quantities in period 2 under asymmetric information are the same as those under perfect information. In the case where $c_B \in \left(c_B^{c'}, c_B^b\right)$, the combined quantity in period 1 of the two products associated with (NB, PL) is obtained by adding (A34) to (A35) and using (A38) to replace μ_1:

$$Q_B^1 + Q_H^1 = \frac{\bar{\theta}}{2} - \frac{c_H}{2s_H} - \frac{(2+\delta)(c_B - c_B^c)\left(c_H - c_H^\gamma\right) - \delta(c_B^a - c_B)\left(c_B^b - c_B\right)}{4s_B s_H c_H}. \qquad (A56)$$

It can be shown that $(Q_B^1 + Q_H^1) - Q_B^*$ is a decreasing function in $c_B \in \left(c_B^{c'}, c_B^b\right)$, and at $c_B = c_B^b$, $(Q_B^1 + Q_H^1) - Q_B^* = \delta(s_B - s_H)(s_H\bar{\theta} - c_H)^2/4s_B s_H c_H > 0$. This implies that $Q_B^1 + Q_H^1 > Q_B^*$ for $c_B \in \left(c_B^{c'}, c_B^b\right)$. Regarding the joint profit associated with (NB, PL) and (NB, 0), we have shown in the proof of Proposition 4 that $\widehat{\Pi}_{BH} > \Pi^*_B$ in this case.

In the case where $c_B \in (c_B^b, c_B^{b'}]$, the combined quantity in period 1 of the two products associated with (NB, PL) is obtained by adding (A40) to (A41) and using (A44) to replace μ_1:

$$Q_B^1 + Q_H^1 = \frac{\bar{\theta}}{2} - \frac{c_H}{2s_H} - \frac{2s_H c_H(c_B - c_B^c) - \delta(s_B - s_H)(c_H^\alpha - c_H)^2}{4s_B s_H c_H}. \qquad (A57)$$

Using (A57) and (14), we can show that $(Q_B^1 + Q_H^1) - Q_B^* = \delta\,(s_B - s_H)(c_H^\alpha - c_H)^2/4s_B s_H c_H > 0$. Regarding the joint profit associated with (NB, PL) and (NB, 0), it is demonstrated in the proof of Proposition 4 that $\widehat{\Pi}_{BH} > \Pi^*_B$ in this case. Q.E.D.

Proof of Proposition 8: We prove this result by comparing the supplier's profits associated with the two product lines. Its profit associated with the product line (NB, 0) is $(1-\omega)\Pi^*_B$. Its profit associated with (NB, PL), on the other hand, depends on the value of c_B. As we have seen in the proof of Proposition 4, for $c_B \in \left(c_B^c, c_B^{c'}\right)$ the equilibrium joint profit is the same as that under perfect information, equaling Π^*_{BH}. Accordingly, the NB supplier's profit in this case is $(1-\omega)\Pi^*_{BH} - (1-\omega)\widehat{\Pi}_H$. Therefore, for $c_B \in \left(c_B^c, c_B^{c'}\right)$ the NB supplier would earn a larger profit from (NB, PL) than from (NB, 0) if $(1-\omega)\Pi^*_{BH} - (1-\omega)\widehat{\Pi}_H > (1-\omega)\Pi^*_B$, *i.e.*, if $\Pi^*_{BH} - \Pi^*_B - \widehat{\Pi}_H > 0$.

From Lemma 2, we have

$$\widehat{\Pi}_H = \frac{(1+\delta)(s_H\bar{\theta} - c_H)^2}{4s_H} - \frac{1}{16s_H}\left\{\frac{(s_H\bar{\theta} - c_H)[(2+\delta)c_H - \delta s_H\bar{\theta}]}{c_H}\right\}^2. \quad (A58)$$

Subtracting (A58) from $\Pi^*_{BH} - \Pi^*_B$ in (A10) and rewriting, we obtain

$$\Pi^*_{BH} - \Pi^*_B - \widehat{\Pi}_H = \frac{(1+\delta)[(s_H c_B - s_B c_H)^2 - s_B(s_B - s_H)(s_H\bar{\theta} - c_H)^2]}{4s_H s_B(s_B - s_H)} + \frac{1}{16s_H}\left\{\frac{(s_H\bar{\theta} - c_H)[(2+\delta)c_H - \delta s_H\bar{\theta}]}{c_H}\right\}^2. \quad (A59)$$

Using (A59), we find that $\Pi^*_{BH} - \Pi^*_B - \widehat{\Pi}_H > 0$ if $c_B > c_B^{c^\dagger}$.

Note that the above comparison of profits is based on the premise that $c_B \in \left(c_B^c, c_B^{c'}\right)$. Hence, the condition $c_B > c_B^{c^\dagger}$ is relevant only if $c_B^{c'} > c_B^{c^\dagger}$. Below we show that $c_B^{c'} > c_B^{c^\dagger}$ if s_B is sufficiently large.

Using (30), we find

$$\lim_{s_B \to \infty}\left(\frac{c_B^{c'}}{s_B}\right) = \bar{\theta}, \qquad (A60)$$

and using (31) we obtain

$$\lim_{s_B \to \infty}\left(\frac{c_B^{c^\dagger}}{s_B}\right) = \frac{c_H}{s_H} + \frac{s_H\bar{\theta} - c_H}{2s_H c_H}\sqrt{\frac{\delta}{1+\delta}[2(2+\delta)s_H\bar{\theta}c_H - \delta c_H^2 - \delta s_H^2\bar{\theta}^2]}\ . \qquad (A61)$$

Combining (A60) and (A61), we find that

$$\lim_{s_B \to \infty}\left(\frac{c_B^{c^\dagger}}{c_B^{c'}}\right) = \frac{c_H}{\bar{\theta}s_H} + \frac{s_H\bar{\theta} - c_H}{2\bar{\theta}s_H c_H}\sqrt{\frac{\delta}{1+\delta}[2(2+\delta)s_H\bar{\theta}c_H - \delta c_H^2 - \delta s_H^2\bar{\theta}^2]} \quad (A62)$$

is less than 1 given that $c_H \in \left(c_H^{\gamma}, c_H^{\alpha}\right)$. Therefore, there exists $S > 0$ such that $c_B^{c'} > c_B^{c^\dagger}$ for $s_B > S$.

The above analysis implies that for a sufficiently large s_B, $\Pi_{BH}^* - \Pi_B^* - \widehat{\Pi}_H > 0$ if c_B is in the interval $(c_B^{c^\dagger}, c_B^{c'}]$. For $c_B \in (c_B^{c'}, c_B^{b'})$, on the other hand, the relevant profit comparison is between $\widehat{\Pi}_{BH} - \widehat{\Pi}_H$ and Π_B^*. Since $\Pi_{BH}^* = \widehat{\Pi}_{BH}$ at $c_B = c_B^{c'}$, by continuity we know that there exists a $c_B^{b^\dagger}$ in the interval $(c_B^{c'}, c_B^{b'})$ such that $\widehat{\Pi}_{BH} - \widehat{\Pi}_H - \Pi_B^* > 0$ if $c_B \in (c_B^{c'}, c_B^{b^\dagger})$. Q.E.D.